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Examining fourth-grade mathematics writing: features of organization, mathematics vocabulary, and mathematical representations

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Abstract Increasingly, students are expected to write about mathematics. Mathematics writing may be informal (e.g., journals, exit slips) or formal (e.g., writing prompts on high-stakes mathematics assessments). In order to develop an effective mathematics-writing intervention, research needs to be conducted on how students organize mathematics writing and use writing features to convey mathematics knowledge. We collected mathematics-writing samples from 155 4th-grade students in 2 states. Each student wrote about a computation word problem and fraction representations. We compared mathematics-writing samples to a norm-referenced measure of essay writing to examine similarities in how students use writing features such as introductions, conclusions, paragraphs, and transition words. We also analyzed the mathematics vocabulary terms that students incorporated within their writing and whether mathematics computation skills were related to the mathematics vocabulary students used in writing. Finally, we coded and described how students used mathematics representations in their writing. Findings indicate that students use organizational features of writing differently across the norm-referenced measure of essay writing and their mathematics writing. Students also use mathematics vocabulary and representations with different levels of success. Implications for assessment, practice, and intervention development are discussed.

Keywords Mathematics · Writing · Fourth grade · Fractions · Word problems

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Introduction

Communication is "an essential part of mathematics," and, for this reason, students need to be provided with instruction on using writing to express mathematical ideas (National Council of Teachers of Mathematics [NCTM], 2000, p. 60). Not only is writing emphasized by the NCTM (e.g., "use the language of mathematics to express mathematical ideas," p. 63), mathematics writing is also emphasized within the *Common Core State Standards* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These standards state that students should be able to communicate precisely to others, construct viable arguments, critique the mathematics reasoning of others, explain how to solve problems, and use clear definitions and vocabulary. In response to NCTM standards and the *Common Core*, new high-stakes assessments require students to use writing to answer and explain mathematics questions.

With these new mathematics writing expectations in mind, we investigated the mathematics writing of fourth-grade students. We asked students to write about two mathematics scenarios: one involving computation with addition, subtraction, and multiplication presented within a word problem and the other involving representations of fractions. Throughout this manuscript, we refer to students' writing about these scenarios as *mathematics writing*. Additionally, we collected data on the essay writing of the students. See Powell and Hebert (in press) for an analysis of the connections among essay writing, computation skill, and mathematics writing. Although students' mathematical skill is important contributor to their ability to write about mathematics (Powell & Hebert, in press), in this manuscript we set out only to examine the mathematics writing provided by students. We describe the features, organization, and mathematical attributes (i.e., vocabulary, representations) included in mathematics-writing samples and make comparisons to performance on a norm-referenced measure of writing.

In this introduction, we discuss the importance of the two mathematical content areas featured within the mathematics-writing tasks: word problems and fractions. Then, we present methods for the measurement of writing quality with an emphasis of mathematics-writing quality. We discuss the importance of written organization and how mathematics vocabulary and representations may be important for mathematics writing. Finally, we present the research questions guiding this study.

Word problems and fractions

In fourth grade, the *Common Core* states that students should "use the four operations with whole numbers to solve problems" (p. 29), and multistep word problems are named specifically. Word problems are the primary method for assessment of mathematics in the late elementary grades. Word-problem solving involves several components, including reading of the problem, planning for solution, setting up for solution, and conducting appropriate computations (Fuchs et al., 2010; Hegarty, Mayer, & Monk, 1995; Jitendra et al., 2013; Parmar, Cawley, & Frazita, 1996). Multistep word problems often cause more difficulty for students than word problems with a single step (Agostino, Johnson, & Pascual-Leone, 2010), but multistep problems provide students with several starting points for problem solution. For this reason, we selected a multistep word problem for one of the mathematics-writing tasks and included the operations of addition, subtraction, and multiplication.

Also in fourth grade, the *Common Core* outlines that students should use "visual fraction models" (p. 30). Strong fraction knowledge is often described a necessary component for algebraic understanding (Booth & Newton, 2012). Visual models of fractions help students understand the magnitude or value of a fraction, which is a difficult concept for students (Bonato, Fabbri, Umlità, & Zorzi, 2007). In the late elementary grades, students should understand fraction models of area, length, and set (Van de Walle, Karp, & Bay-Williams, 2013), but there is variability in fraction understanding among the models (Charalambous & Pitta-Pantazi, 2007). Because fraction understanding is vital for upper-level mathematics and because many students only interpret fractions using one model, we selected a fraction task with multiple visual representations for the other mathematics-writing task.

Measuring writing quality

Assessment of student performance on general writing tasks (e.g., descriptive, expository, narrative, opinion, and persuasive) is well researched (e.g., Correnti, Matsumura, Hamilton, & Wang, 2013; Donovan & Smolkin, 2011; Graham, Berninger, & Fan, 2007; Jeffery, 2009; Olinghouse, 2008; Peterson, Childs, & Kennedy, 2004). Overall, the assessment of general writing indicates many factors contribute to writing quality. The two most common ways of measuring writing quality are analytic and holistic (Graham, Harris, & Hebert, 2011). Scoring writing quality analytically, as is used in 6 + 1 traits writing, includes scoring for elements such as ideas, organization, voice, word choice, sentence fluency, conventions, and presentation (Culham, 2003). When analyzing writing quality analytically, teachers and researchers commonly score writing against a rubric for each of the traits. Quality may also be assessed holistically by an analysis of reasoning, evidence, and idea development (e.g., Correnti et al., 2013; Kim, Al Otaiba, & Wanzek, 2015). Often, holistic scoring involves rating students' writing samples on a numerical scale, with a rubric, or comparing to benchmark papers (Beck & Jeffery, 2007; Jeffery, 2009; Graham, McKeown, Kiuhara, & Harris, 2012). For general writing prompts, writing may also be assessed using curriculum-based measures examining specific features: total number of words written, number of paragraphs, length of sentences, correctly spelled words, correct and incorrect word sequences, vocabulary utilized, punctuation, or handwriting (Coker & Ritchey, 2010; McMaster & Espin, 2007).

Comparing writing samples within a simple genre can be difficult unless raters agree on specific metrics. Moreover, comparing writing quality across genres can be even more difficult due to the constraints and expectations that different genres place on the writer. Previous research indicates performance in one genre may not be reflective of writing skill in other genres (Engelhard, Gordon, Walker, & Gabrielson, 1994; Graham et al., 2011; Popp, Ryan, Thompson, & Behrens, 2003).

Using generalizability theory, Graham, Hebert, Sandbank, and Harris (2016) determined that 11 different compositions were needed to reliably estimate the across-genre writing achievement of second- and third-grade students. This may be reflective of the differences in elements and skills required for writing in by different genres.

In addition to qualities and features of writing, cognitive models of writing suggest that writers engage in complex tasks involving planning, organizing, goal setting, reading, editing, audience awareness, self-regulation, and the integration of content and discourse knowledge (e.g., Bereiter & Scardamalia, 1987; Flower & Hayes, 1981; Harris, Graham, MacArthur, Reid, & Mason, 2011). As students learn to write across different genres, some of the complexities of writing tasks may be similar. There may, however, be important differences to consider when writing in different genres, especially when content specific knowledge is needed, such as in the genre of mathematics writing.

Mathematics writing

Empirical research about mathematics writing is limited. A handful of studies indicate student understanding of mathematics concepts and procedures may improve with the incorporation of mathematics-writing activities (e.g., Bicer, Capraro, & Capraro, 2013; Cross, 2009; Kostos & Shin, 2010). In none of these studies, however, did the researchers describe the quality of the mathematics writing or specifics of mathematics content that may contribute to quality mathematics writing.

While not based on empirical work, several educators provided suggestions about mathematics writing. Ediger (2006) stated that mathematics writing must have a purpose. Burns (2004) provided strategies such as asking students to brainstorm and share ideas before writing, providing prompts for mathematics writing, and discussing important vocabulary. McCarthy (2008) suggested teachers use a graphic organizer. With this organizer, students write a topic sentence in the middle, write three reasons to support the topic sentence, and write one sentence to support the topic. Students draw pictures to represent each box and then write a story based on the graphic organizer. Verlaan (2009) also supported the use of graphic organizers. These suggestions for mathematics-writing activities from other educators may be helpful for encouraging teachers to help students with mathematics writing, yet these suggestions have not been investigated within experiments specific to mathematics writing. As mathematics writing is a combination of mathematics and writing, it is necessary to understand how students organize mathematics writing.

Organization of writing

The organization of writing varies widely across genres according to the elements of the genre. For example, persuasive writing includes elements such as topic sentences, reasons, elaborations, counter-arguments, and conclusions, while story writing included elements such as characters, settings, events, endings, and emotions (Mason, Harris, & Graham, 2011). Despite this, some organizational characteristics of writing might be more consistent. For example, introductions, conclusions, paragraphs, and transition words may be elements seen in persuasive writing, argument writing, and informational writing. Because very little empirical research has been conducted to examine the features of mathematics-writing organization, it is important to examine whether and how students organize mathematics writing in relation to other writing genres, as well as how they incorporate mathematics specific content, such as mathematics vocabulary and mathematical representations, into their writing.

Mathematics vocabulary

Mathematics vocabulary refers to written words that express mathematical concepts or procedures, and mathematics vocabulary is necessary for demonstration of mathematics proficiency (NCTM, 2000; Riccomini, Smith, Hughes, & Fries, 2015). Monroe and Panchyshyn (1995) described mathematical vocabulary as falling into one of four categories. Vocabulary can be (1) technical with one meaning only applicable in mathematics. Vocabulary can be (2) subtechnical with multiple meanings that vary across content areas or within mathematics (e.g., *degrees* of an angle versus *degrees* of temperature). Mathematics vocabulary can also be (3) general vocabulary used in everyday language or reading (e.g., *shade* or *find*). Vocabulary may also be (4) symbolic, meaning it is represented using numerals or symbols (e.g., *$* or *three*). Other research teams used Monroe and Panchyshyn's mathematics vocabulary categories for organizing types of mathematics vocabulary (e.g., Powell & Driver, 2015; Harmon, Hedrick, & Wood, 2005; Pierce & Fontaine, 2009).

In terms of a focus on mathematics writing and vocabulary, at the university level, Stonewater (2002) asked students in calculus classes to write about mathematics. Stonewater determined that higher scoring writers used mathematical vocabulary and appropriate notation and symbols, whereas lower scoring mathematics writers made irrelevant comments about mathematics and used mathematical notation incorrectly. Little is known, however, about how elementary students use mathematics vocabulary terms in writing. In fact, much of the literature related to mathematics vocabulary at the elementary level is not evidence-based and merely suggestions for approaches to teaching mathematics vocabulary. Most suggestions are based on acquisition strategies taught in reading. In one empirical study, Monroe and Pendergrass (1997) learned that students who learned mathematics vocabulary using graphic organizers, rather than definitions, understood mathematics vocabulary better at posttest. Monroe and Pendergrass did not evaluate how students used mathematics vocabulary in mathematics writing.

In an overview of the mathematics vocabulary presented in the glossaries of three mathematics curricula across kindergarten through eighth grade, Powell (2016) identified over 800 distinct mathematics vocabulary terms. Each term could be identified as falling into one or more of Monroe and Panchyshyn's (1995) categories of mathematics vocabulary (i.e., technical, subtechnical, general, or symbolic). Given that students are responsible for knowing and understanding the meaning of hundreds of mathematics vocabulary terms, and because it is necessary for students to understand mathematics vocabulary in order to demonstrate competency mathematics (Schleppegrell, 2007), we aimed to learn how students write with mathematics vocabulary with a focus on mathematics vocabulary used in mathematics writing. We wanted to understand which mathematics vocabulary terms students used in writing and how students utilized mathematics vocabulary in mathematics writing.

Mathematics representations

In mathematics, teachers often use multiple representations to help teach a mathematics concept. These multiple representations may include the use of hands-on manipulatives (i.e., concrete), pictorial representations that match the concrete manipulatives (i.e., representational), and solving problems with numerals and symbols (i.e., abstract). This way of providing multiple ways to access mathematics is termed the concrete-representational-abstract framework (CRA; Miller & Hudson, 2006). The CRA framework is a renaming of Bruner's (1966) modes of representation (i.e., enactive, iconic, and symbolic). For example, for solving a problem such as 19 + 27, a student could use concrete Base-10 blocks to show tens and ones, the action of addition, and any regrouping of 10 ones into one 10. A student could also draw or use virtual pictorial representations to show tens and ones, the action of addition, and any regrouping. These different ways to representing 19 + 27 help the student understand the concept of addition and establish a procedural algorithm (e.g., partial sums, traditional algorithm) for problem solution. Ultimately, students use numerals and symbols to solve 19 + 27.

Because CRA is quite prevalent in mathematics research (e.g., Mancl, Miller, & Kennedy, 2012; Moreno, Ozogul, & Reisslein, 2011) and as students are regularly expected to interpret mathematics using pictorial representations in textbooks and on high-stakes assessments (van Garderen, Scheuermann, & Jackson, 2012), we tracked whether students used pictorial representations or abstract numerals and symbols in their mathematics writing. If students used pictorial representations, expressions, or equations to support their writing, we acknowledged these alternative ways (i.e., not using written words).

Purpose and research questions of the present study

As the current collection of research related to mathematics writing is not based on empirical research, we aim to develop and test an intervention related to developing the mathematics-writing skill of students in the late elementary grades. In order to develop an efficacious intervention related to teaching mathematics writing, it is necessary to understand which qualities of mathematics writing may be necessary for mathematics writing proficiency. As previously stated, mathematics skill and instruction are important to consider in any future mathematics-writing intervention. We limit the scope of the current manuscript, however, to the examination of writing elements. Specifically, the present study aims to inform specific components of writing instruction that may contribute to future mathematics-writing interventions through the analysis of students' use of organizational features, mathematics vocabulary, and mathematical representations.

Our research questions were as follows:

1. Do the organizational features of students' essay writing compare to the organizational features they employ in mathematics writing? That is, does students' mathematics writing compare in length (number of words), and do they incorporate the use of introductions, conclusions, paragraphs, novel transition expressions into mathematics writing? Does the use of these general writing features differ based on the mathematics-writing prompt content?
2. What mathematics vocabulary do students use when writing about mathematics? Specifically, which mathematics vocabulary terms do students use frequently?
3. How do students use representational (i.e., pictures) and abstract (i.e., equations, expressions) representations in mathematics writing? That is, how do students support their mathematics writing with pictures and equations?

Method

Participants

We sampled participants ($N = 155$) from eight, 4th-grade classrooms in a Plains state and a Southwestern state. The average age of students was 10 years, 4 months, and 73 students were male. In terms of race/ethnicity, five students were African American, five were Asian, 17 were Hispanic, 125 were White, and three were categorized as other. In the sample, 24 students had a school-identified disability. In the Plains district, 44.2 % of students qualified for reduced and/or free lunch. In the Southwestern district, 18.9 % of students qualified for reduced and/or free lunch.

Measures

All 155 students completed three measures: essay writing, mathematics writing about word problems, and mathematics writing about fractions.

Essay writing

To measure students' essay writing ability, we administered the Essay Composition subtest of the *Wechsler Individual Achievement Test—third edition* (WIAT-III, Psychological Corporation, 2009). On the Essay Composition subtest, the examiner read the prompt aloud, and then students had 10 min to write about their favorite game and provide at least three reasons why the game is their favorite. We scored Essay Composition utilizing a rubric of theme development and text organization provided by the WIAT-III. Students earn up to 2 points for a thesis statement in the introduction, up to 2 points for a conclusion statement, and 0–5 points for the number of paragraphs. A paragraph was defined as having at least two punctuation marks and being separated using line spacing or indentation. Students also earned 0–5 points for each novel transition expression following punctuation (e.g., another, second, finally). Students earned 0–3 points for each reason of why they like a game and an additional 0–3 points for an elaboration for each reason. Maximum score was 20. Following WIAT-III instructions, we also counted the total number of separate words written. Median reliability, as reported by Breaux (2010), is .85 for fourth-grade students in the spring.

Word-problem mathematics writing

We assessed mathematics writing using a researcher-developed Math Writing Word Problem (MW-WP) prompt (see Fig. 1). We designed this prompt to assess computation skill within a word-problem scenario as fourth-grade mathematics standards adopted by most states in the United States require students to solve "multistep word problems…using the four operations" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 29). In the MW-WP prompt, a hypothetical student, "Sam," solved a word problem in four steps. Step A was unnecessary and involved multiplication of single-digit numbers. In step B, Sam added two-digit numbers ($20 + $20), but this step was also unnecessary. Step C involved addition of monetary values, and Sam made a regrouping mistake in the ones place. Step D required subtraction, and Sam used an incorrect minuend and subtrahend. Sam also lined up the numbers for computation incorrectly and arrived at an incorrect answer. To administer the prompt, the examiner read the prompt aloud. Then, students wrote for 10 min.

We developed a scoring system for MW-WP based on the scoring rubric for WIAT-III Essay Composition. Students earned up to 2 points for a thesis statement in the introduction, up to 2 points for a conclusion statement, and 0–5 points for the number of paragraphs. Students also earned 0–5 points for each novel transition expression. In terms of mathematics content, students earned points for each of the steps of Sam's work, such as identifying correct numbers and operation, elaborating mistakes, and providing correct answers. As the mathematics portions of the scores are not addressed in this manuscript we do not go into detail about the mathematics portions of the scores here. See Powell and Hebert (in press) for more detail about scoring. Maximum score for MW-WP was 34, and Cronbach's α was this sample was .73. We also counted the total words written. We counted each numeral (e.g., 4, 9, 20) and symbol (e.g., $, +, =) as a unique word. We also counted each mathematics vocabulary term used, whether the student used equations or expressions in their writing, and whether the student drew pictorial representations as part of their mathematics writing.

Fractions mathematics writing

We assessed writing about fractions using a researcher-developed Math Writing Fraction (MW-FR) prompt (see Fig. 2). This prompt aligned with fourth-grade mathematics standards about "using visual fraction models" for representing

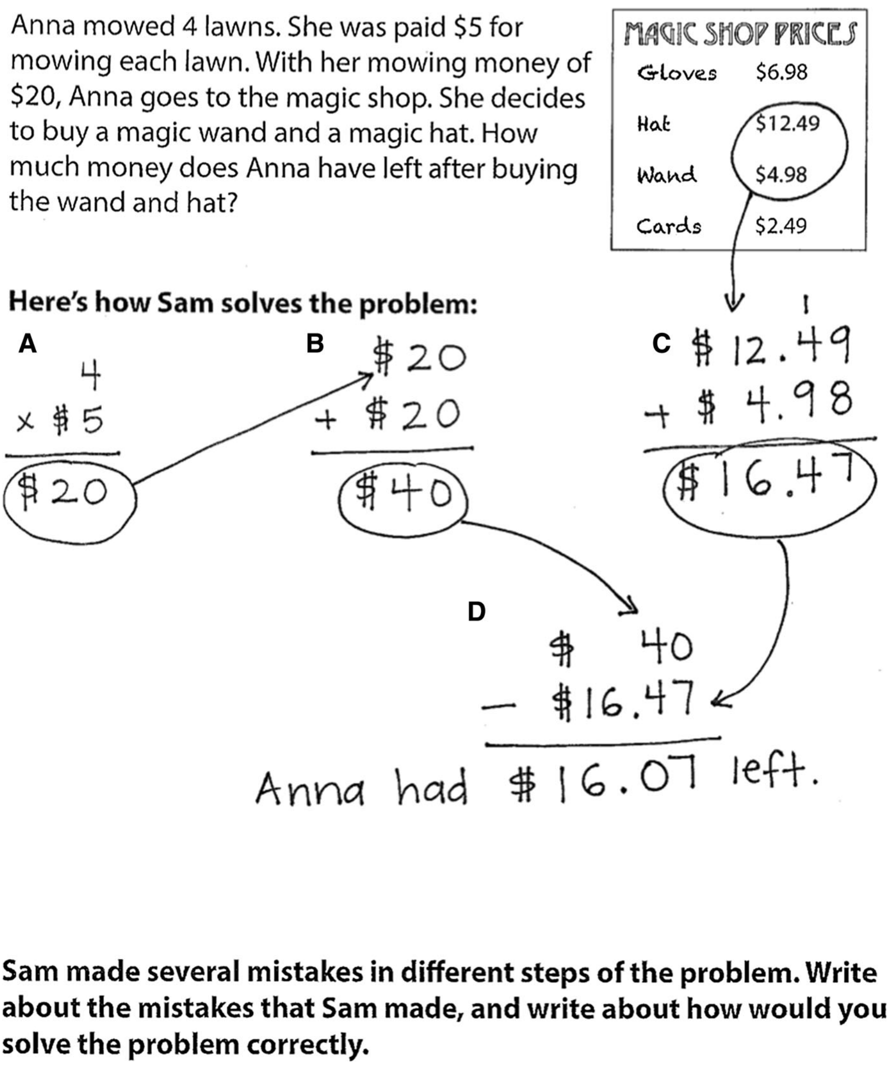

Fig. 1 Math Writing Word Problem (MW-WP) prompt

fractions (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 30). We used the three models of fractions (i.e., area, length, set; Van de Walle, Karp, & Bay-Williams, 2013). In the MW-FR prompt, a hypothetical teacher asked four students, Alex, Bo, Cole, and Deb, to draw the fraction three-fifths. Alex drew a rectangular area model with five unequal sized boxes; three were shaded. Bo drew a length model with eight equal sized boxes; three were shaded. Cole drew a circular area model with two circles, and each circle was divided equally into thirds. Five parts were shaded. Deb drew a set model with five circles; three circles were shaded. To administer the prompt, the examiner reviewed the prompt, and then students wrote for 10 min.

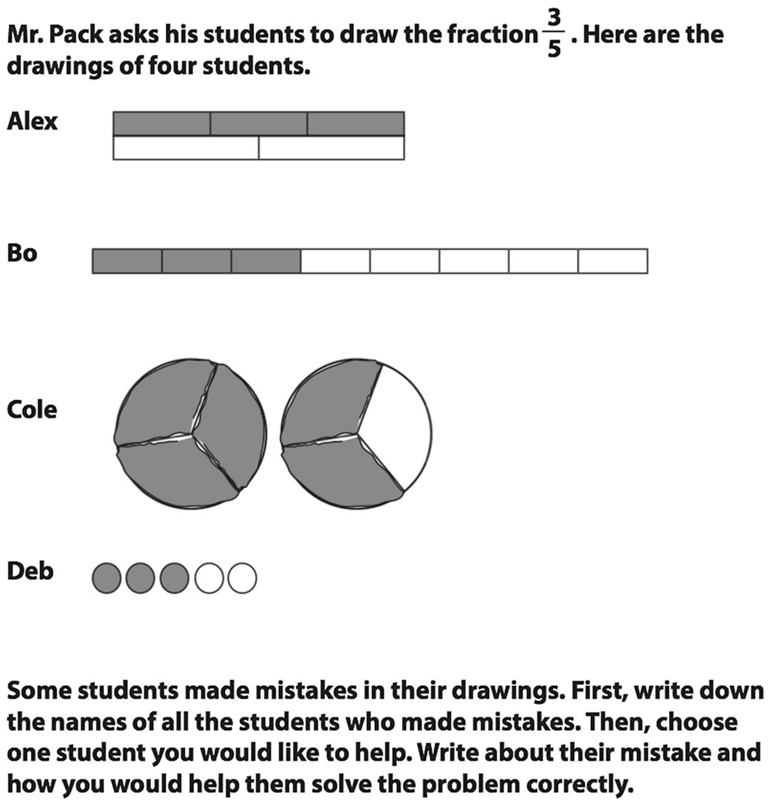

Fig. 2 Math Writing Fraction (MW-FR) prompt

Similar to the MW-WP, we developed and scored MW-FR based off of the Essay Composition of the WIAT-III. MW-FR scoring for introduction, conclusion, paragraphs, and transition words was identical to MW-WP. For mathematics content, students earned points to identifying which students made mistakes, and for providing corrections for mistakes (see Powell & Hebert, in press, for more information on the mathematics content scoring). Maximum score for content was 38, and Cronbach's α for this sample was .72. Additionally, we counted the total words and symbols written, instances of mathematics vocabulary, and instances of pictorial representations.

Inter-rater reliability

Five raters (the two authors and three graduate students in education) scored the assessments and assisted in creating the database. Two scorers experienced with the WIAT-III rubric scored the Essay Composition writing samples for all students. For training purposes, the raters first double-scored a random sample of 10 writing samples. Following training, one rater scored all of the remaining writing samples, and a second rater scored a random sample of 25 % of the essays for reliability purposes; inter-rater reliability, calculated as the percentage of agreement, was 90 %. Only the scores from the first rater were used in the analyses.

For MW-WP and MW-FR, the two authors trained together and double scored the writing samples from one classroom and resolved disagreements through discussion. Because these were newly developed assessments, scoring standards were developed and agreed upon during training. Next, the two authors each acted as the primary rater for half of the remaining writing samples. A graduate assistant then acted as a secondary rater and scored a random sample 20 % of the essays for reliability purposes; inter-rater reliability, calculated as the percentage of agreement, was 92 % for MW-WP and 94 % for MW-FR.

Procedure

Testing occurred in two, 30-min whole-class testing sessions administered in the late spring of the school year. WIAT-III Essay Composition was administered in the first session. MW-WP and MW-FR were administered in the second session in a counterbalanced order across classrooms. Examiners were the two authors (with graduate degrees in special education) and a graduate student with a Master's degree in school psychology. All examiners were trained to administer the measures by following the same test administration procedures and reading from the same test administration script.

Analyses

Statistical comparisons were made between students' scores of features of writing organization on their essay writing and mathematics-writing tasks, including introductions, conclusions, number of paragraphs, number of transitions, and number of words. Analyses were conducted using STATA/SE 12 (StataCorp, 2011). For each model, we used contrast codes as level-1 variables to (1) determine whether there were differences on the outcome variables between essay writing and both math-writing conditions combined and (2) determine whether there were differences on the outcomes variables between MW-WP and MW-FR. For contrast one (essay writing compared to both mathematics-writing samples), essay writing was coded as 1, and both mathematics-writing samples were coded as $-.5$. For contrast two (MW-WP compared to MW-FR), essay writing was coded as 0, MW-WP was coded as -1, and MW-FR was coded as 1. We also included classroom as a level-2 variable to control for classroom-level variance in all models. Logistic, Poisson, and linear regression models were estimated based on the nature of the specific outcome variable examined.

As previously stated, *introductions* and *conclusions* were both originally scored on a three-point scale (e.g., $0 =$ no introduction, $1 =$ introduction with an incomplete these statement, and $2 =$ introduction with complete thesis statement). Because the scores for these variables were limited in range, we collapsed scores of 1 and 2 into a single score, allowing us to examine the variable using a logistic regression model. In other words, *introductions* and *conclusions* were both simplified into absent (0) or present (1). With outcomes reshaped into a long format, we used the xtmelogit command in STATA/SE 12 (StataCorp, 2011) to examine the odds of students including an introduction or conclusion in each type of writing using the following multi-level mixed effects logistic regression model.

Because *number of paragraphs* and *number of transitions* were both count variables, we used Poisson regression models to compare students' use of paragraphs and transition words across the writing types. With outcomes reshaped into a long format, we used the xtmepoisson command in STATA/SE 12 (StataCorp, 2011) to examine number of words using the following multi-level mixed effects Poisson regression model.

Although *number of words* could also technically be considered a count variable (a count of the number of words), it functions more like a continuous variable due to the large number of words and few scores bounded at zero. With outcomes reshaped into a long format, we used the xtmixed command in STATA/SE 12 (StataCorp, 2011) to examine number of words using the following multi-level mixed effects linear regression model.

Results

The results for the features of writing are presented in three categories, in alignment with the research questions: organizational features, mathematics vocabulary, and symbols and pictorial representations. For an examination of the overall scores for essay writing and mathematics writing see Powell and Hebert (in press), as these results are beyond the scope of this manuscript.

Organizational features of writing

The organizational features of writing included introductions, conclusions, number of paragraphs, number of transition words, and total number of words. Means and standard deviations for the organizational features can be found in Table 1. The regression models for each of the outcome variables are displayed in Table 2.

Table 1 Means and standard deviations of measures

Variable	WIAT-III Essay		MW-WP		MW-FP	
	M	(*SD*)	*M*	(*SD*)	*M*	(*SD*)
Introduction	0.73	(0.49)	0.03	(0.16)	0.59	(0.86)
Conclusions	0.15	(0.39)	0.08	(0.28)	0.05	(0.25)
Paragraphs	1.36	(0.90)	0.86	(0.51)	0.86	(0.69)
Transition words	1.08	(1.20)	0.81	(1.08)	0.24	(0.55)
Number of words and symbols	98.10	(36.69)	59.03	(31.09)	62.94	(32.88)

WIAT-III Wechsler Individual Achievement Test (3rd ed.), *MW-WP* Math Writing Word Problem, *MW-FR* Math Writing Fractions

Table 2 Results from multi-level models contrasting essay writing, MW-WP, and MW-FR across features of writing organization

	Outcome				
	Introductions[a]	Conclusions[a]	Paragraphs[b]	Transitions[b]	Words[c]
Fixed effects					
Intercept	−1.139***	−2.454***	−0.003	−0.528***	73.056***
Essay versus mathematics writing[d]	2.092***	0.581*	0.305***	0.596***	24.576***
MW-WP versus MW-FR[e]	1.541***	−0.331	−0.004	−0.614***	5.658**
Random effects					
Classroom ($n = 8$)	0.47	0.24	0.07	0.14	8.21

MW-WP Math Writing Word Problem, *MW-FR* Math Writing Fractions

* $p < .05$; ** $p < .01$; *** $p < .001$

[a] Multi-level mixed effects logistic regression

[b] Multi-level mixed effects Poisson regression

[c] Multi-level mixed effects linear regression

[d] Contrast coded writing types: Essay = 1; MW-FR = −.5; MW-WP = −.5

[e] Contrast Coded writing types: Essay = 0; MW-FR = 1; MW-WP = −1

Introductions

The use of introductions in each of the writing samples was compared to determine whether there was an association between students' use of introductions and conclusions across the writing types. Because introductions were recoded as a binary variable, multi-level mixed-effects logistic regression was used to analyze this writing feature.

The results of the model indicate the coefficient was significant and negative for the comparisons of essay writing to both math-writing types ($p < .001$), indicating that the odds of students including an introduction in their essay writing was about 2.59 times more likely in essays than in mathematics writing. Additionally, the coefficient for the contrast between MW-WP and MW-FR was also significant ($p < .001$) and positive, indicating the odds of students including introductions in their mathematics writing was 1.6 times greater in MW-FR than in MW-WP. Additionally, there was a significant classroom effect, indicating that use of introductions varied by classroom.

Conclusions

Conclusions allow writers to summarize the main points of their writing and provide any final interpretations, analysis, or comments. Because conclusions were recoded as a binary variable, we again used multi-level mixed-effects logistic regression to analyze this writing feature.

Students used very few conclusions in all three writing samples. Despite that, the results of the model indicate the coefficient was significant and negative for the comparisons of essay writing to both mathematics-writing types ($p = .01$), indicating that the odds that students included conclusions in their essay writing was 1.8 times greater than in their mathematics writing. The coefficient for the contrast between MW-WP and MW-FR, however, was not significant, indicating there was no significant difference in the odds that conclusions were used across the two mathematics-writing types. There was a significant classroom effect, indicating that use of conclusions varied by classroom.

Number of paragraphs

Paragraphing is an organizational feature of writing that allows the writer to shift between important points and group ideas related to overarching points, while distinguishing those ideas from other overarching points being made. The average number of paragraphs used by students in their essay writing was greater than one, while the number of paragraphs in both mathematics-writing samples was less than one. A writing sample with less than one paragraph indicates that students are not using punctuation and sentence conventions correctly when constructing a written response, as paragraphs, defined by our scoring procedures, were required to have at least two sentences grouped together with appropriate punctuation. Because number of paragraphs is a count variable, multi-level mixed-effects Poisson regression was used to analyze this writing feature.

The results of the model indicate the coefficient was significant and negative for the comparisons of essay writing to both mathematics-writing types ($p < .001$), indicating that students were likely to use paragraphs more often in their essay writing than in their mathematics writing. On the other hand, the coefficient for the contrast between MW-WP and MW-FR was not significant ($p = .951$), indicating there was no difference in the number of paragraphs students wrote in their mathematics-writing samples. There was a significant classroom effect, indicating that use of paragraphs varied by classroom.

Transition words

Writers use transition words to indicate transitions between topics across sentences and paragraphs within a writing sample. Because number of paragraphs is a count variable, multi-level mixed-effects Poisson regression was used to analyze this writing feature. The results of the model indicate the coefficient was significant and negative for the comparisons of essay writing to both mathematics-writing types ($p < .001$). This result indicates that students used significantly more transitions in their essay writing than in their mathematics writing. Additionally, the coefficient for the contrast between MW-WP and MW-FR was significant and negative ($p < .001$), indicating that students used significantly more transitions in MW-WP than in MW-FR. There was a significant classroom effect, indicating that use of transition words varied by classroom.

Number of words

Number of words may be less of an organizational feature than a content generation and fluency feature. We included it in the organization section because it provided an indicator of the amount of content students needed to organize. In other words, it is possible that written responses with fewer words also require fewer organizational features to be included by the writer. In this comparison, the total number of words and symbols were used for the mathematics-writing samples, as we thought this would best represent the best comparison of total content generated across each writing sample.

The results of the model indicate the coefficient was significant and negative for the comparisons of essay writing to both mathematics-writing types ($p < .001$), indicating that students wrote significantly more words in their essay writing than in their mathematics writing. Additionally, the coefficient for the contrast between MW-WP and MW-FR was significant and positive ($p = .002$), indicating that students wrote significantly more words in MW-FR than in MW-WP. Similar to the other results, there was a significant classroom effect, indicating that the number of words written by students varied by classroom.

It is likely that the additional cognitive complexity of analyzing and interpreting the mathematics problems (e.g., identifying mistakes, recalculating mathematics problems, or drawing pictorial representations) left students with less time to write words. However, it is worth noting that many students completed their written responses before the 10-min writing time allotted for the responses expired.

Mathematics vocabulary

To address our second research question, we coded use of mathematics vocabulary, symbols, equations, and pictorial representations. See Table 3 for averages of mathematics vocabulary instances for MW-WP and MW-FR according to the Monroe and Panchyshyn (1995) categorization of mathematics vocabulary.

Table 3 Mathematics vocabulary

Variable	MW-WP		MW-FR	
	M	(*SD*)	*M*	(*SD*)
Number of words	44.46	(26.71)	58.08	(30.89)
Number of symbols	14.57	(10.51)	4.86	(3.89)
Number of words and symbols	59.03	(31.09)	62.94	(32.88)
Mathematics vocabulary	21.95	(12.24)	15.21	(9.14)
Technical	2.90	(2.39)	2.35	(2.31)
Subtechnical	0.91	(1.16)	1.59	(1.86)
Symbolic (number)	7.98	(4.99)	4.79	(3.81)
Symbolic (word)	0.48	(1.38)	1.04	(1.78)
Symbolic (symbol)	6.59	(5.95)	0.06	(0.32)
General	3.08	(2.78)	5.37	(4.30)

MW-WP Math Writing Word Problem, *MW-FR* Math Writing Fractions

Table 4 Most prevalent mathematics vocabulary terms

Category	Term	MW-WP Instances	MW-FR Instances
Technical	Add/added	183	10
	Subtract	112	3
	Fraction	0	88
	Circle/circles	0	66
	Rectangle/rectangles	0	59
	Dollar	45	0
	Money	41	0
	Square/squares	0	36
	Number/numbers	19	13
	Denominator	0	23
Subtechnical	Part/parts	0	47
	Equal	11	34
	Whole	5	25
	Total	11	18
	Line/lines	0	26
	First (not transition)	22	3
	Bar	0	25
	Regroup	21	0
	Carry	21	0
	Even/evenly	0	18
Symbolic (number)	20	242	0
	5	54	148
	3	2	176
	3/5	0	143
	40	122	0
Symbolic (word)	One	27	57
	Three	1	49
	Two	18	28
	Five	3	36
	Twenty	17	0
Symbolic (symbol)	$ (dollar sign)	602	0
	+ (plus sign)	134	5
	= (equal sign)	107	5
	– (minus sign)	89	0
	× (multiplication sign)	38	0
General	Wrong	94	97
	Shade/shaded/shading	0	138
	Answer	97	17
	Right	50	60
	Piece/pieces	0	83
	Correct/correctly	34	35

Table 4 continued

Category	Term	MW-WP Instances	MW-FR Instances
	Color/colored	0	68
	Left	55	7
	Draw/drew/drawn	0	60
	Top	24	11

Prevalent determined as 10 terms with the most instances in each of the Technical, Subtechnical, and General categories; 5 terms with the most instances in each of the Symbolic subcategories

MW-WP Math Writing Word Problem, *MW-FR* Math Writing Fractions

For each mathematics-writing sample, we calculated the total number of words written (i.e., words with letters) and the total number of symbols written (e.g., $, 20). Numbers, such as 20, $^3/_8$, or 2.53, were coded as a single term because the number represents one amount. The totals were combined for a sum of words and symbols written.

From the total number of words and symbols written, we categorized the mathematics vocabulary terms. To do this, mathematics vocabulary was broadly defined; that is, any term referring to mathematics in some manner was coded into the spreadsheet. We had to use a broad definition of mathematics vocabulary to capture how students wrote about mathematical concepts. For example, we included *top* as a mathematics vocabulary term as students use this term instead of *numerator* (e.g., *top* number) or to describe *regrouping* (e.g., place the one on *top*). In a similar way, we coded the terms *carry* and *borrow*, which are procedural descriptors for *regroup*, but not mathematically correct. We also coded terms such as *shade*, *color*, *bar*, and *pieces*, which students used frequently to describe fractions. Without context, many of these terms would not read as mathematical. Altogether, we coded 115 novel mathematics words and 96 novel mathematics numbers and symbols. For any term, we coded plurals and changes in tense under a single category (e.g., *draw*, *drew*, and *drawn* were categorized as one mathematics vocabulary term).

After coding the instances of use for each mathematics vocabulary term, we categorized all terms as technical, subtechnical, general, or symbolic, according to Monroe and Panchyshyn's (1995) categories of mathematics vocabulary. To understand nuances in student writing about numbers and symbols, we used subcategories (i.e., number, symbol, word) within the symbolic strand. The number category represented a number presented with numerals (e.g., 3, 20.00), the symbol category indicated a mathematics symbol (e.g., +, =), and the word category represented a written number word (e.g., *twenty*, *one*). On average, students used mathematics vocabulary terms for about one-third of the total words for MW-WP writing. On the MW-FR, approximately one-fourth of words were mathematics vocabulary terms.

For the MW-WP prompt, students used symbolic numbers in their mathematics writing most often. This was followed by symbolic symbols. Instances of general terms were slightly more prevalent than use of technical terms, whereas subtechnical terms and symbolic words were written infrequently. For the MW-

FR prompt, the pattern of mathematics vocabulary use differed. General terms were written most often, following by symbolic numbers. Technical and subtechnical terms were written at approximately the same rate. Symbolic symbols were rarely written.

For a qualitative look at the specific mathematics vocabulary terms students used within their MW-WP and MW-FR writing (see Table 4), we indicated the most used mathematics vocabulary terms for each of the four vocabulary categories. Difference emerge base on the mathematics content of the prompt. For example, *subtract* was used 112 times in the MW-WP prompt and only 3 times in the MW-WP prompt. Students often used shapes to describe fraction models, and specific terms that show a strong understanding of a fraction (e.g., *denominator*, *numerator*) were used infrequently. *Wrong*, *correct*, and *right*, all general terms, appeared in MW-WP and MW-FR writing samples with approximately the same number of instances. Symbolic symbols were quite prevalent in MW-WP writings. The dollar sign ($) was written approximately four times as often as any other symbol. Symbolic numerals were used frequently in both mathematics-writing prompts, and symbolic words were used more often in MW-FR than MW-WP writings.

Mathematics representations

In addition to coding for mathematics vocabulary, we also examined how students used mathematical equations and pictorial representation in their writing. Not surprisingly, pictorial representations were used only for the fraction problem writing and equations were used only in the word problem writing, as the use of these representations was context specific. In each case, we coded for whether the representations were used: (1) as a replacement for writing, (2) as an elaboration or extension of writing, or (3) as a component that fits within the writing as a replacement for some content, but also as an example at the same time. We also coded for whether the representation was used correctly or incorrectly.

Use of equations

We included this categorization separately from mathematics vocabulary symbols, as multiple symbols were used within the equation or expression to provide a complete understanding of a procedure. Fifty-three students (34 %) used equations in their MW-WP writing. Each instance of equations use was double coded by the authors for categorization purposes.

When students wrote equations, they used equations in their writing as replacements for writing (83 %), incorporated the equations within their writing (81 %), used them to restate part of the prompt (53 %), used them to show their work (21 %), or used them to elaborate on their written explanations (2 %). We also coded for whether students correctly set up their problems when using them in writing, and performed the correct calculations. Only 32 % of student set the problem up correctly, with many students writing the numbers for subtraction problems in the wrong order. Despite this, 52 % of the students performed the calculation correctly, demonstrating that the students understand the function of the equations but do not understand how to set them up correctly in writing.

Using pictorial representation

In the MW-FR problem, the prompt included pictorial representations of the fractions. Therefore, we expected students to provide pictorial representations of the fractions in their writing samples. However, only 16 students (10 % of the sample) included a pictorial representation within their writing.

When students included pictorial representation of the fractions in their writing, they used them as elaborations of their written explanations (81 %), used them as a replacement for written explanations (12.5 %), used them to provide a re-representation of the prompt (12.5 %), and incorporated them within the written paragraph (12.5 %). Some students used pictorial representation in multiple ways. Students represented the drawing correctly 81 % of the time and incorrectly 19 % of the time. Interestingly, every correct representation of the drawn fraction was a bar or rectangle model, while all of the incorrect representations were circle representations.

Discussion

In this study, we aimed to explore features of mathematics writing to describe how fourth grade students organize mathematics writing, use mathematics vocabulary and symbols, and provide mathematics representations.

Organizational features of writing

When comparing writing quality of the MW-WP and MW-FR writing samples to the WIAT-III Essay Composition, statistically significant differences emerged in all areas. Students included more introductions and conclusions, wrote more paragraphs, used more transition words, and wrote more words in their essay writing than in the mathematics-writing samples. This may suggest that students have a better understanding of how to organize their essay writing than mathematics writing, are more predisposed to organize their essay writing due to length, or do not feel the need to organize their mathematics writing in the same way. However, there may be specific considerations for each organizational feature for specific writing types.

Regarding introductions, whether students wrote an introductory statement may be indicative of whether the prompt encouraged such a statement. For example, in the essay-writing example, students are first asked to write about their favorite game, and then include three reasons why they like it. Because of this, students may be predisposed to include an introduction about the favorite game. Similarly, in the MW-FR example, students were explicitly prompted students to write the names of students who made mistakes and then to select one student to help. Students began MW-FR with statements like, “Today I will be helping Cole.” However, the MW-

WP prompt encouraged students to write about the mistakes of hypothetical student Sam. Because the prompt implicitly encouraged students to begin writing without an advanced organizer, only a few students wrote introductory statements such as "Sam had some mistakes in his problems."

Because of the timing aspect of all three writing prompts, very few students had the opportunity to write conclusion statements. One student wrote, "Finally, you are done…that said, Anna had $2.53 left," but many other students left sentences unfinished. With more time or unlimited time, more students may have had the opportunity to generate and write conclusion statements. Under ideal conditions, mathematics writing would be untimed, but in a collection of pilot data, we put time limit on the writing to make it comparable to the standardized writing task. Also, writing assessments are typically administered under timed conditions (e.g., Correnti et al., 2013; Espin, Weissenburger, & Benson, 2004; Hall-Mills & Apel, 2015).

In terms of the number of paragraphs, students wrote fewer paragraphs on the mathematics-writing tasks than on the WIAT-III Essay Composition. The use of symbols, equations, and pictorial representations may have led to more mistakes in sentences writing and fewer complete sentences, leading to fewer paragraphs being counted. Additionally, we hypothesize that checking or performing mathematical computations (or drawing pictorial representations before writing) may have contributed to students not having as much writing time on the MW-WP and MW-FR tasks. The total number of words and symbols written indicates that students wrote significantly less than on the MW-WP and MW-FR than on the WIAT-III. Less writing time would also contribute to fewer paragraphs. Also, students were unfamiliar with writing about mathematics, so they may have not transferred essay writing skills (e.g., organizing information into paragraphs) to mathematics-writing tasks.

The number of transition words varied by mathematics-writing prompt. MW-WP was set up to encourage transition words as the hypothetical student, Sam, solved the problem in steps. Steps easily convert to transition words, such as "*Next*, Sam messed up in step C," or "*Lastly*, she got step D wrong." Several students used transition words on the MW-FR task (e.g., "*Now*, I will help Bo."), but this occurred less often than on the MW-WP task.

For future intervention development, we would aim to provide explicit instruction related to transfer of essay writing skill to mathematics writing. We learned that even when students have a minimum level of proficiency with essay writing features, this proficiency may not translate to mathematics writing. Differences emerge based on the content of the mathematics writing. We also believe that students may need more time to complete mathematics-writing prompts due to the combination of analyzing and solving a mathematics problem(s) within the mathematics-writing task.

Mathematics vocabulary

To understand how students utilized mathematics vocabulary to write about mathematical concepts and procedures, we coded instances of mathematics vocabulary terms. One out of every three or four written words and symbols had a mathematical connotation. The majority of these instances were symbolic, with students writing symbols, numbers, and, on fewer occasions, drawing pictorial representations. Several students used symbols as words. The student who wrote "You added 20 + 20 to = 40" used the plus and equal signs instead of writing *plus* and *equal*. In a similar way, a student wrote "He forgot to—$20—16.47," with the minus sign used instead of the term *subtract*. For written words, most mathematics vocabulary terms were essay and procedural in nature. For example, when explaining incorrect subtraction on the MW-WP prompt, a student wrote "he shouldn't have put $40 over the cents." This is a procedural explanation and does not elaborate upon why dollars and cents cannot be lined up for computation based on place value. Another student, writing about the same mistake, wrote "he didn't line up his numbers, when you correctly line up your numbers and subtract…" to help correct the mistake, but this also does not indicate why the mistake of the hypothetical student is incorrect. For explanation of the mistake, one student wrote "he's acting like $40 is 40¢." Another procedural explanation that students often wrote about was related to regrouping. Many students included a sentence such as "Sam forgot to carry." This definition of regrouping is procedural, not conceptual, and is not favorable for instruction on regrouping concepts.

Some students did write about properties of mathematics. For example, "Alex did 3/5 but they are not even," "All of her pieces were not the same size," or "It wasn't fairly divided." Here, each student discussed a fundamental concept of fractions: all parts of a fraction must be equal parts. Other students used proper terminology (e.g., "Cole made an improper fraction."). One student's definition, "the denominator means total spaces and the numerator means total shaded in" is accurate and conceptual. Another student's example included a procedural definition with the similar-sounding term for *denominator*, "Denomination (the number on the bottom)." When describing how to help the hypothetical students on MW-FR correct their mistakes, students often provided procedural descriptions, such as "write one circle and draw 5 lines and color 3 shapes," or a student would write "This is how I do it" and draw a pictorial representation.

Results from our second research question, that is the type of mathematics vocabulary, frequency of use, and accuracy of use, may inform future intervention development related to mathematics writing. Essay writing research indicates that important vocabulary terms are typically seven letters or more in length (Graham et al., 2007). This guideline would not work well in mathematics as the majority of important mathematics vocabulary comprises fewer than seven letters (e.g., *more*, *less*, *three*, *divide*, *add*, *equal*, etc.). Future research about mathematics-writing assessment and how to provide effective instruction on mathematics writing may need to provide explicit instruction on the meaning and use of mathematics vocabulary terms, especially those terms necessary for explanations and elaboration. For example, teaching students to use *numerator*, *denominator*, and *equal parts* for mathematics writing about fractions and *regroup*, *add*, *subtract*, and *multiply* for mathematics writing about computation. These vocabulary terms would lead to mathematically accurate writing, instead of *do*, *carry*, *borrow*, *split*, and *color*.

Mathematics representations

Although most students used symbols in their writing (89 % of the writing samples across both prompts included symbols), the results indicated a large majority of students did not include equations or pictorial representations. It may be that it is more efficient to refer to specific elements of word problems or fraction representations than it is to include the entire equation. For example, students who identified a mistake in an equation might need to refer to the specific aspect of the equation in which the mistake occurs (e.g., "Sam did not regroup when adding the 9, 4 and 1," or "Sam did not line up the 40 with the correct place values for 16.47 in the subtraction problem"), rather than providing the entire equation. Similarly, students who wrote an explanation about the numerator and denominator of the fractions may not have deemed it necessary to include a pictorial representation. In fact, the prompt did not ask for one to be included in the written answer.

Teachers and researchers need to determine whether teaching students to include equations and pictorial representations within their writing is appropriate and desired. When students did include equations and pictorial representations, students used the equations and pictures to illustrate their understanding (or misunderstandings). For example, it was clear that many students did not know how to set up equations properly in their writing. Quite a few students placed the numbers for subtraction in the incorrect order, which would lead to a negative integer for an answer (if the subtraction was performed correctly). Similarly, we noted that all of the fraction representations drawn using circles were incorrect, as opposed to the representations using bars and rectangles, which were all correct. Teaching students how to include mathematical representations in the form of equations and pictorial representations may help students elaborate on their explanations written in words, allowing teachers to assess conceptual understandings more easily. It may also be a more efficient way to write about mathematics in some instances.

We suggest including equations and pictorial representations is an important skill to teach in any mathematics-writing intervention. Based on the data from this study, some students attempted to include these representations in their writing, but often unsuccessfully. Mathematics teachers, writing teachers, and researchers should be consulted to determine whether and when it would be appropriate to include equations and pictorial representations in mathematics writing. This is completely new territory because, as far as we can hypothesize, it is not necessary or possible to utilize equations or pictures in essay writing prompts for other genres. How the inclusion of these mathematical elements affects mathematics writing scores needs to be investigated in future research. Intervention could then be designed to teach mathematics content and context appropriate knowledge for when, how, and why to include mathematics equations and representations.

Limitations

We note several limitations to the present study. First, we administered all three writing prompts under timed conditions. The WIAT-III Essay Composition is a timed measure, so we used similar timing restrictions for both MW-WP and MW-FR. An untimed measure may have provided students with more planning time and more time to conduct necessary mathematics computation before writing. With an untimed measure or more time (e.g., 15 min, 20 min), we may see more use of conclusion statements and an increase in the number of words written and paragraphs written.

Second, MW-WP and MW-FR are not directly comparable because the prompts differed between these two measures. MW-WP prompted students to write about the mistakes of a hypothetical student on several steps of a word problem; the MW-FR prompt encouraged students to select one student to help on a fraction representation. The nature of these prompts may have led students to use more transition words or organizational features in one task over the other. Future research should focus on creating and evaluating mathematics-writing prompts that are comparable and standardized.

Third, our measure of essay writing ability (i.e., WIAT-III Essay Composition) represented one genre of writing (i.e., essay writing). Other genres could provide meaningful information about student writing ability that might be compared with mathematics writing (Gillespie, Olinghouse, & Graham, 2013). Additionally, we only administered one measure of essay writing ability. Huang (2008) indicated that three writing tasks should be collected in order to demonstrate a score about writing ability, whereas Graham and colleagues (2016) reported that 5–11 separate writing tasks were needed.

Fourth, our sample of participants was not diverse. Despite conducting assessments in two distinct regions of the United States, the populations were primarily English speaking Caucasian students without disabilities. Future research needs to be conducted with students with disabilities and difficulties, as well as English learners.

Fifth, self-efficacy and motivation may significantly impact students' performance on mathematics-writing tasks, as both mathematics and writing are cognitively demanding tasks. Although examination of these factors was outside the scope of the research questions for this paper, future research should be conducted to determine the influence of these factors on students' mathematics writing.

Finally, we did not collect data on the curricula used in the classrooms from which we collected data. Although we were able to analyze the mathematics writing of students, we do not know how much instruction students have received in this skill area. Before the development of any intervention can occur, more research should be conducted to examine the mathematics writing that occurs in classrooms, as well as how the instruction may influence students' mathematics writing.

Implications for research and practice

Additional research is necessary in the area of mathematics writing, especially as more high-stakes assessment include mathematics items that require written responses (e.g., PARCC, 2015). We need to develop and pilot mathematics-writing prompts with prompts that generate similar writing style across mathematics content areas. Researchers should investigate the effect of allowing time for pre-writing or analysis of mathematics (Read, 2010). Other lines of research could investigate whether students write more when encouraged to use or not use pictorial representations and equations, expressions, or inequalities.

Researchers should develop mathematics-writing tasks in other mathematics content areas. We selected word problems and fractions because of the emphasis of this content at fourth grade, but mathematics writing in other areas (e.g., decimals, multiplication, geometry, algebra) could elicit different organizational writing features from students, and thus, different outcomes. As more mathematics-writing prompts are developed, scoring rubrics that accompany each prompt must be developed and scrutinized. It may be helpful to analyze rubrics from mathematics-writing tasks on high-stakes assessments to gather information about determinants of high-quality mathematics writing.

In terms of practice, we conducted this study to gather data qualitative information about mathematics writing as it relates to essay writing and mathematics attributes. As high-stakes assessments encourage students to write about mathematics, and because the our results indicate wide variability in writing quality as it relates to mathematics writing, interventions may be necessary to improve the mathematics writing of students. The current landscape of research about mathematics writing provides suggestions for mathematics-writing activities, but no empirical work has investigated necessary and effective components of mathematics-writing intervention at the general or special education levels. As indicated by the procedural mathematics vocabulary terms that students used in their mathematics writing, intervention cannot be focused on teaching students to write about mathematics in strictly a procedural manner. As we examined, mathematics writing also includes writing organization and idea development, as well as correct mathematics work. If students complete mathematics-writing interventions, researchers should also investigate whether improved mathematics writing positively influences essay writing ability.

Conclusion

In sum, we learned that features of essay writing (e.g., introduction, conclusion, paragraphs, transition words) vary among the two mathematics-writing tasks and the task of essay writing. Some of this variability may be due to the wording of the prompt; other variability may be due to the nature of the mathematics embedded within the mathematics-writing prompts. In terms of mathematical attributes, students use mathematics vocabulary terms in their writing, although use is often procedural and relies heavily on symbols. Some students used equations or pictorial representations to support their mathematics writing. Future research should include a deeper investigation of assessment of mathematics writing using a combination of essay writing features, mathematical vocabulary, equations, and pictorial representations.

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