

# **Three-Tier Mathematics Assessment & Intervention Model**

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**This model is based on the 3-Tier Reading Model  
Vaughn Gross Center for Reading & Language Arts**

# Advance Organizer

**•Legislation:  
IDEA 2004**

**•Mathematics 3-Tier  
Tiers  
Assessment  
Instruction**

# **Individuals with Disabilities Education Act (2004) (reauthorization)**

Provides emphasis on prevention and intervention

Contains provision to use scientific, research-based interventions as part of the process to determine eligibility for learning disabilities

Continues to require that lack of instruction must be considered in the eligibility decision-making process

Provides provision for LEAs to use identification approaches as an alternative to the discrepancy model → response to intervention (RTI)

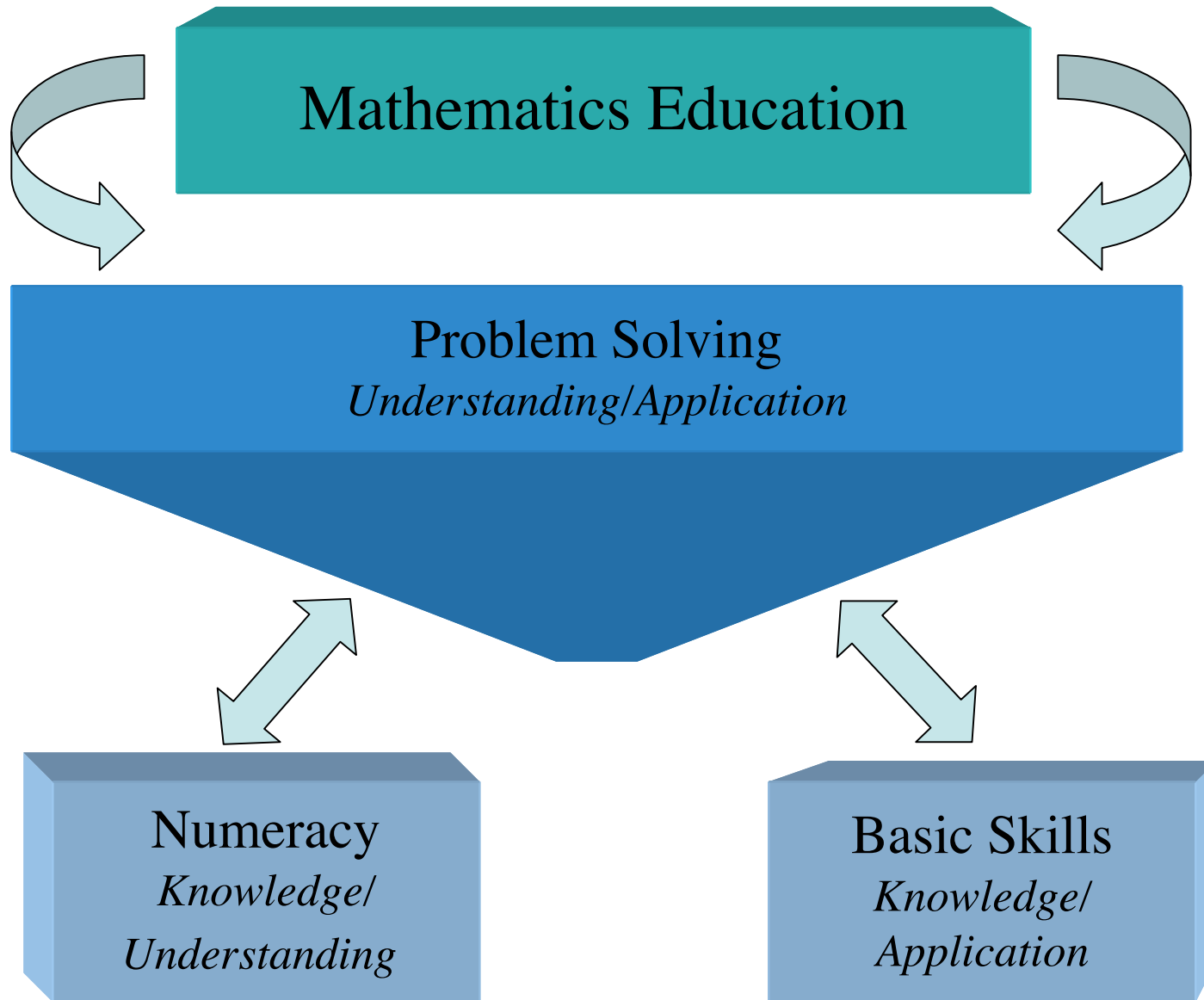
# Core Features of RTI

1. High quality classroom instruction that is research-based -  
Can be determined by examining the research base of the programs being used  
Can be assessed by comparing students' learning rates and achievement across same grade level classrooms
2. Universal screening on academics and behavior - Criteria are used to judge the learning and achievement of all students
3. Continuous progress monitoring - Data can be used to determine students who are not reaching benchmark
4. Research-based interventions - Possibilities include standard protocol procedures that have been validated  
May occur 8 - 12 weeks in length; Designed to be more intensive
5. Fidelity measures -Documentation that procedures are being implemented accurately as described and validated through research (observational checklist of critical teaching behaviors)



## What is the 3-Tier Mathematics Model?

- Framework:**
  - Provides a framework for adapting general education class instruction, intervening with struggling students, and using assessment data to inform instructional decision-making.
- Assessment:**
  - Designed to assess student pre - post abilities on mathematics (TEKS)
  - Designed to identify/diagnose students who are “slow starters” in kindergarten and who demonstrate mathematics difficulties in number & operation
  - Designed to determine student response to intervention
- Mathematics Standards:**
  - Focuses on standards-based interventions (number & operation).
- Instruction/Intervention:**
  - Tier 1: Adaptations
  - Tier 2: Intervention in number & operation, algebra, problem solving
  - Tier 3: Developing



# What do the NCTM Principles & Standards (2000) tell us?

## Prekindergarten - Grade 12 Standards:

- **Content Standards: content knowledge & skills students should learn**
  - Number & Operations
  - Algebra
  - Geometry
  - Measurement
  - Data Analysis & Probability
- **Process Standards: ways of acquiring & using content knowledge**
  - Problem Solving
  - Reasoning & Proof
  - Communication
  - Connections
  - Representation

# **NCTM Curriculum Focal Points & Connections, Sept. 2006**

<http://www.nctm.org/focalpoints/downloads.asp>

## **Kindergarten:**

**Number & Operations: Representing, comparing, and ordering whole numbers and joining and separating sets**

## **First Grade:**

**Number & Operations & Algebra: Developing understandings of addition & subtraction and strategies for basic addition facts and related subtraction facts**

**Number & Operations: Developing an understanding of whole number relationships including grouping in tens & ones**

## **Second Grade:**

**Number & Operations: Developing an understanding of the base-ten numeration system and place-value concepts**

**Number & Operations & Algebra: Developing quick recall of addition facts and related subtraction facts & fluency with multidigit addition and subtraction**

## Challenges Before Us: Tier 1 Core: SBRR/SBMR

<b>Reading</b>	<b>Mathematics</b>
<b>Established scientifically-based reading research</b>	<b>NSF Funded Research Supporting Reform Efforts Inquiry Based</b>

## More Challenges Before Us: Tier 1 Core Program

<b>Reading</b>	<b>Mathematics</b>
<b>Established core reading program criteria and programs</b>	<b>Need for core math program criteria</b>

## More Challenges Before Us: Instructional Time

<b>Reading</b>	<b>Mathematics</b>
<b>Established</b> <b>Tier 1: Throughout the day</b> <b>Tier 2: 30 minutes</b> <b>Tier 3: two 30-minutes</b>	<b>Divergent</b> <b>Tier 1: Throughout the day</b> <b>Tier 2: ?</b> <b>Tier 3: ?</b>

## More Challenges Before Us: Instructional Groupings

<b>Reading</b>	<b>Mathematics</b>
<b>Established</b> <b>Tier 1: Flexible</b> <b>Tier 2: Homogeneous</b> <b>1:3, 1:4, 1:5</b> <b>Tier 3: Homogeneous</b> <b>1:3</b>	<b>Divergent</b> <b>Tier 1: Flexible</b> <b>Tier 2: ?</b> <b>Tier 3: ?</b>

## More Challenges Before Us: Tier 2 & 3 Content and Instruction

<b>Reading</b>	<b>Mathematics</b>
<b>Established five core areas of reading; Explicit instruction</b>	<b>NCTM Standards, Focal Points; Explicit Instruction; Secondary &amp; Tertiary Needed</b>

## More Challenges Before Us: Assessment

<b>Reading</b>	<b>Mathematics</b>
<b>Established tests for Screening Diagnosis Outcomes Progress Monitoring</b>	<b>Some established, some emerging tests for Diagnosis Outcomes Progress Monitoring</b>

## **Core Educational Problem: Assessment**

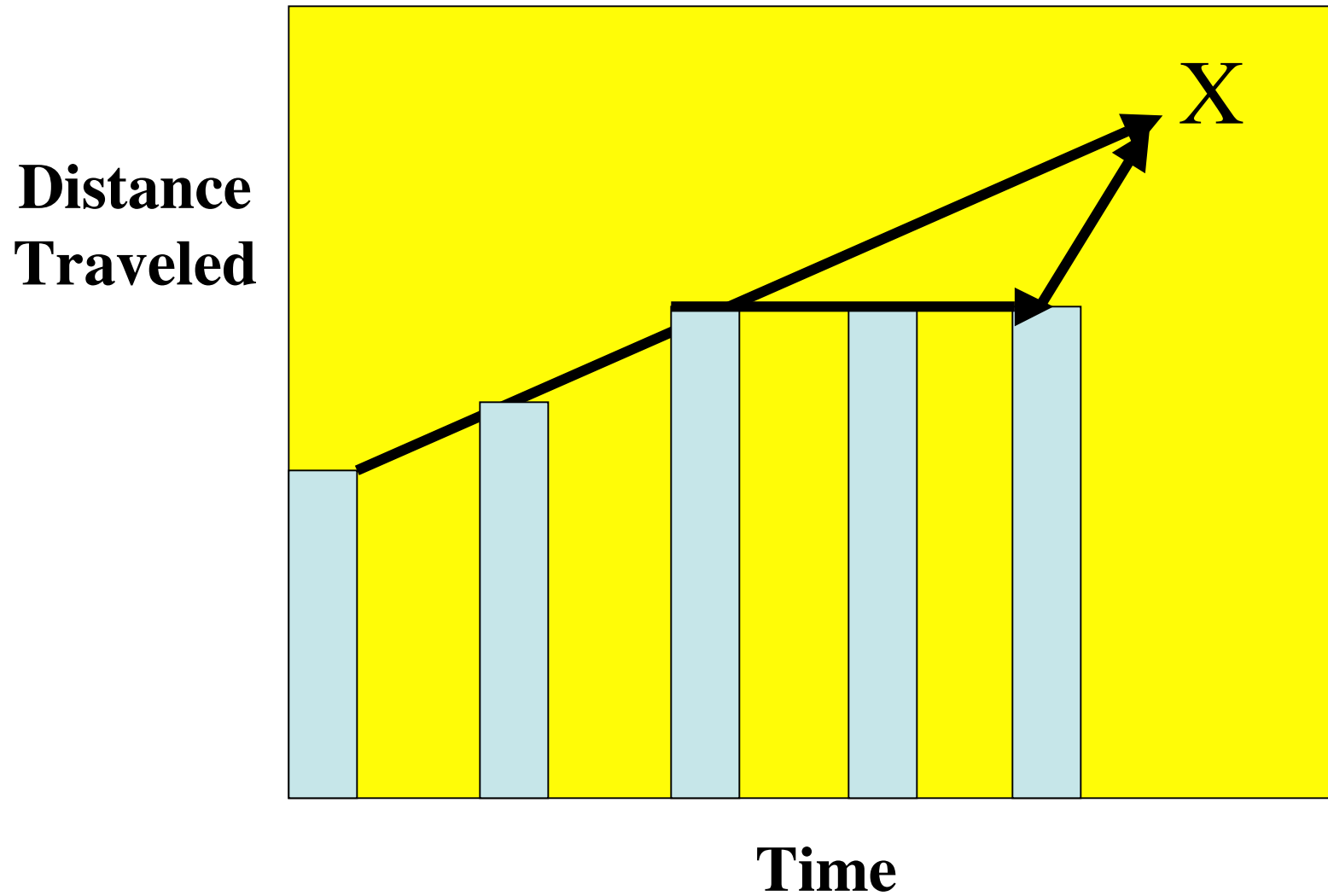
- Limited availability of technically adequate measures for identification and to monitor response to intervention of Tier 2 students in the primary grades**
- Need to develop technically adequate measures for early mathematics number, operation, and quantitative reasoning skills and concepts**
- Measures can contribute to an understanding of predictors of early mathematics performance, inform mathematics instructional decisions, and change mathematics outcomes for students who are at risk for mathematics difficulties**

(Chard, Clarke, Baker, Otterstedt, Braun, & Katz, 2005)

## **When are Assessments Conducted in the 3-Tier Mathematics Model?**

- ❑ Diagnostic/Progress Monitoring: Numeration and Operations; IDs Tiers 2 & 3; intervention progress monitoring**
- ❑ Probes (created/in development): Emphasis on numeration and operation; Bi-weekly (fortnight) Booster Probes and Clinical Interviews**
- ❑ Outcome: Beginning and end of year for all students; broad based skills (Mathematics Problem Solving [K-2] and Mathematics Computation [1&2])**

# Back on Track



# Levels of Progress Monitoring

progress monitoring: a set of techniques for assessing student performance on a regular and frequent basis (*R. Quenemoen, M. Thurlow, R. Moen, S. Thompson, A. Blount Morse*)

What was learned today?  
*Independent Practice*

**Activity Level**

What was learned over 2 weeks; was it maintained and can it generalize to a testing format?  
*Booster Probes*

**Maintenance/Generalization Level**

What was learned this month, and can it generalize to the larger instructional content?  
*Monthly testing using Forms B, C, and D of Content Measures*

**Content I Level**

What was learned this trimester?  
*Pre-, Mid-, and Post-testing using Form A of Content Measures*

**Content II Level**

# *Content PM*

## *Measures*

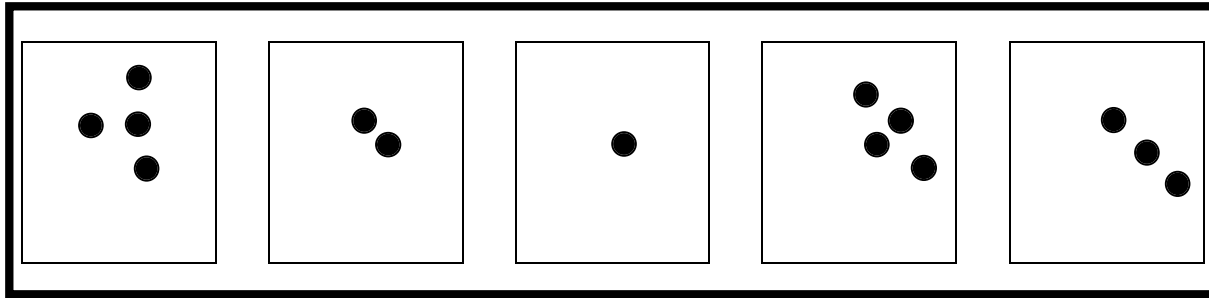
■ ■	1	2	3	4
■	1	2	3	4
■ ■ ■ ■ ■ ■ ■	5	6	7	8

**Number Identification (K  
only) 1 - 20**

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# *Content PM Measures*



**Quantity Recognition (K Only)**  
**1-6**



# *Content PM Measures*

## **Magnitude Comparisons**

**K: 0-20, bigger #, same**

**1: 0-99, smaller #, same**

**2: 0-999, less, equal**



1	0	9	4	1	1	14	10
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# *Content PM Measures*

## **Numeric Sequences**

**K: 0-20**

**1: 0-99**

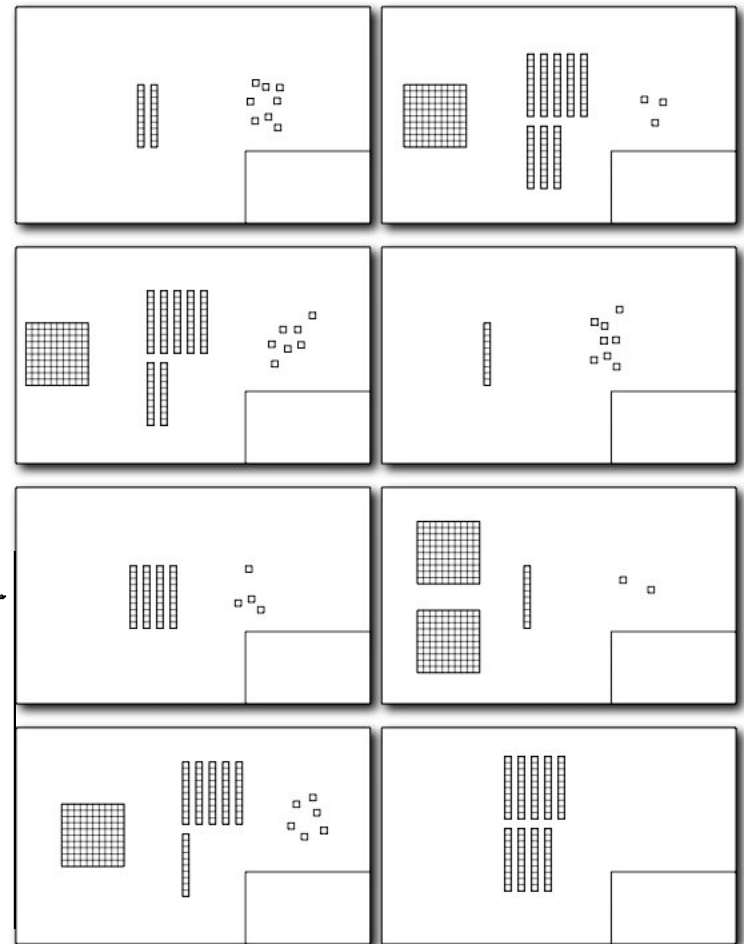
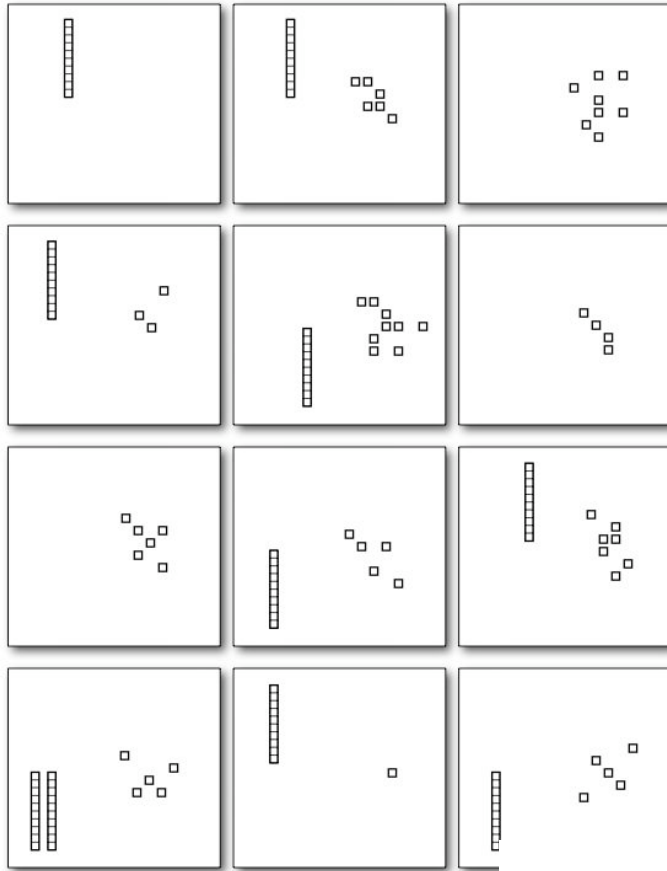
**2: 0-999**



1 2 _____	16 _____ 18	_____ 81 82
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# Place Value

## *Content PM Measures*



**1: 1-99**  
**2: 1-999**



# *Content PM*

## *Measures*

**Addition/Subtraction  
Combinations, to/from 18  
(1 & 2 only)**



$\begin{array}{r} 6 \\ - 2 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ + 1 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ - 1 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ - 8 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ + 0 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ + 3 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ - 2 \\ \hline \end{array}$
$\begin{array}{r} 4 \\ + 3 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ - 6 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ + 0 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ - 7 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ - 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ - 0 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ + 2 \\ \hline \end{array}$

# Purpose 1: Cut-scores and Identification

**Total Score = Sum of subtest standard scores  
converted to SS (Mean = 100, SD = 15)**



**<16<sup>th</sup> %ile**

**“Diagnosis of Struggling Students”**

# Probes and Clinical Interviews

9	6	4	7	5
<u>+9</u>	<u>+6</u>	<u>+4</u>	<u>+7</u>	<u>+5</u>
1	8	3	2	1
<u>+1</u>	<u>+8</u>	<u>+3</u>	<u>+2</u>	<u>+1</u>
8	7	2	4	1
<u>+8</u>	<u>+7</u>	<u>+2</u>	<u>+4</u>	<u>+1</u>
3	5	6	9	0
<u>+3</u>	<u>+5</u>	<u>+6</u>	<u>+9</u>	<u>+0</u>

**“Identify Specific Instructional Content”**

**“Maintenance/Generalization Level Progress Monitoring”**

# What have we learned?

## Findings from Focus Groups & Observations On Tier 1

- **TEKS-based math instruction for typically achieving and low achieving students**
- **Core instruction based on state standards, National Science Foundation instructional recommendations, NCTM *Standards* (teacher created lessons, basal-based instruction)**
- **Time**
  - **Average 90 minutes**
- **Groupings**
  - **Whole/small grouping**
  - **Mixed-level grouping**
  - For low achieving students**
    - **More small grouping for supplemental instruction**
- **Materials & manipulatives**
  - For low achieving students**
    - **More use of manipulatives**

## What else have we learned?

### ▪ **Instructional practices**

#### ▪ **For typically achieving students**

- **Modeling/examples**
- **Math center activities**
- **Practice opportunities**
- **Use of math journal**
- **Cognitively Guided Instruction (CGI)**

#### ▪ **For low achieving students**

- **Supplemental instruction (one-on-one teaching, reteaching & remodeling)**
- **Immediate feedback**
- **Guided practice opportunities**
- **Explicit instruction**

## What else have we learned?

- **Adapted instruction for low achieving students**
  - **Small/peer-mediated instruction**
    - **One-on-one instruction/reteaching**
    - **Changing difficulty level**
    - **Use of various strategies, materials & tasks**
    - **Error correction/immediate feedback**
  - **Assessment in TEKS-based math instruction**
    - **Daily assessment (checking for understanding/observation)**
    - **Teacher-developed check lists**
    - **Chapter tests**
    - **Use of District benchmarks**
    - **Student's math journal**

# What have we learned?

## Basal Analysis

- **Background: Utilization of textbooks**
  - **Classroom emphasis on curricular materials**
  - **Effects of textbooks on instruction and learning**
  - **Design of instructional features**
- **Purpose: Examine the extent to which K-2 mathematics basal programs adhered to the components of effective instructional design**
- **Textbooks: EM=Everyday Math; H-B=Harcourt-Brace; SF=Scott Foresman; SRA=Science Research Associates**

# Results

Criteria	EM	H-B	SF	SRA	% Fully Met Criteria
1. Clarity of objective	3	3	2.67	3	75%
2. Additional skills/concepts taught	1.67	2.67	2.33	2	0%
3. Use of manipulatives & representation	2.67	2.67	2.33	2.67	0%
4. Instructional approach	1.67	2.33	2	2.33	0%
5. Teacher examples/guided practice	2	3	2.67	2.33	25%
6. Independent practice opportunity	2	3	3	2	50%
7. Review of requisite math skills	2.33	1	3	1.67	25%
8. Error correction & corrective feedback	2	1	3	1	25%
9. Vocabulary	2.33	2.33	2.33	1	0%
10. Strategy	2.33	1.67	1.33	1.67	0%
11. Progress monitoring of the lesson	1.33	1.67	2.67	2	0%
<b>% Fully Met Criteria</b>	<b>9.09%</b>	<b>27.27%</b>	<b>27.27%</b>	<b>9.09%</b>	<b>18.18%</b>

# What components should be part of Tier 1 instruction to help at-risk students access the general education curriculum?

- Balanced approach to mathematics instruction
- Opportunities for students to engage in meaningful practice  
Partner Math    CAI
- Mathematically enriched environment that makes math visible  
Vocabulary Words    Abstract Symbols  
Manipulatives and Tools    Calculators
- Explicit instruction to teach procedural knowledge (basals)
- Questioning strategies that require explanations and descriptions
- Progress monitoring
- Problem solving
- Instructional adaptations

# *ADAPT*

**A**sk yourself, “What is the task that I am expecting my students to accomplish?”

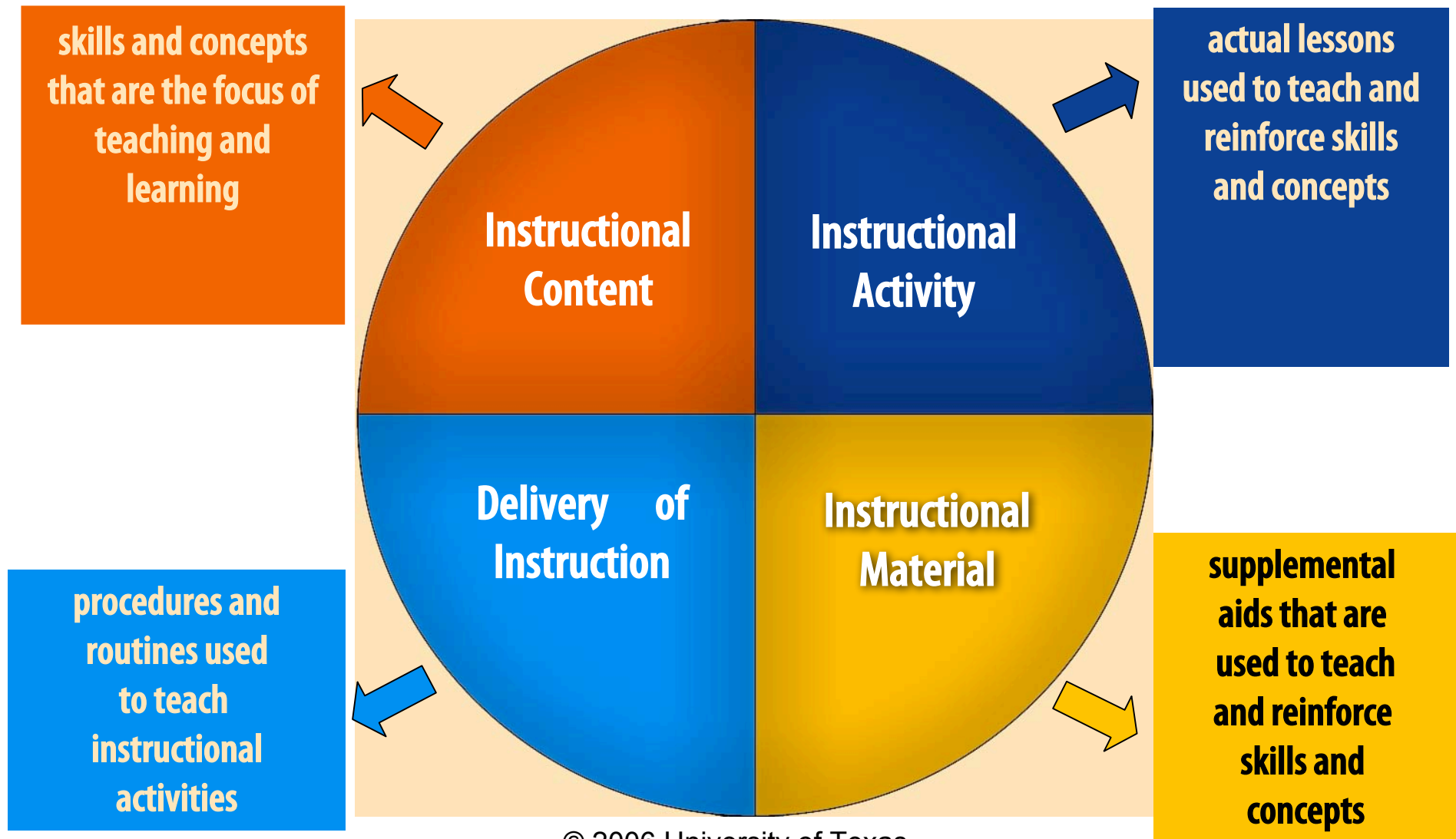
**D**etermine the prerequisite skills needed to accomplish the task.

**A**nalyze the student’s strengths and struggles as they relate to the prerequisite skills.

**P**ropose and implement adaptations from the 4 categories to help the student accomplish the tasks.

**T**est to determine if the adaptations helped the student accomplish the task.

# 4 Categories of *ADAPT*ations



# **Core Educational Problem: Intervention**

**Limited evidenced-based interventions demonstrating efficacy for improving mathematics performance in early mathematics skills and concepts**

**Need to develop, refine, and evaluate interventions to teach students in kindergarten, first, and second grades who have been identified as Tier 2 for mathematics difficulties**

**Number and operations is cited as the most important area of NCTM's (2000) *Principles and Standards for School Mathematics* (Clements & Sarama, 2004)**

**Automaticity is identified as “desirable” at an early stage of formal mathematics education (Cumming & Elkins, 1999)**

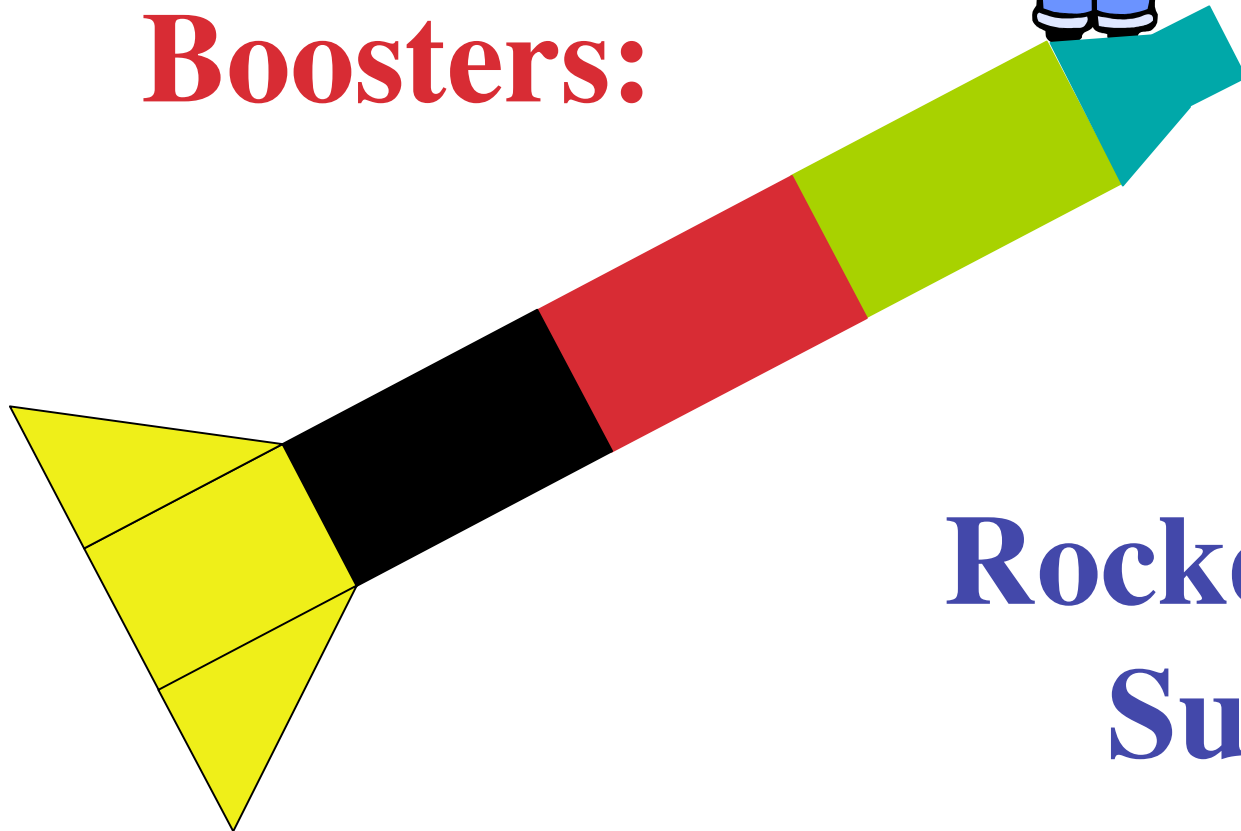
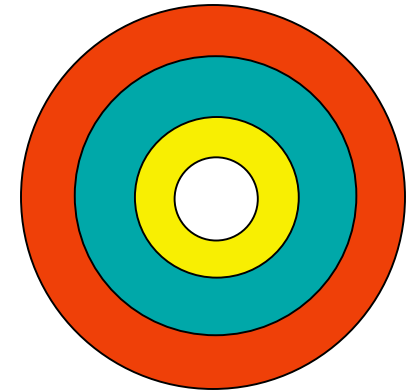
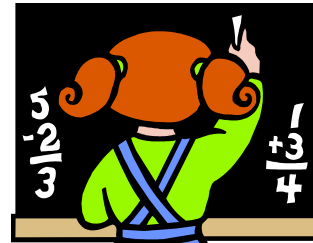
## **Tier 2 and 3 Instruction**

**Small group intervention for students performing at below the 16th (and 10th) percentile.**

- **Boosters**
- **Incorporate the critical features of effective instruction**

*Intervention...*

**Mathematics  
Boosters:**



**Rocketing To  
Success**



**Progress Monitoring: % correct addition**

**Instruction Booster 3: Addition w/counting on 2s**

**Fluency Booster 2: Numeric Sequencing**

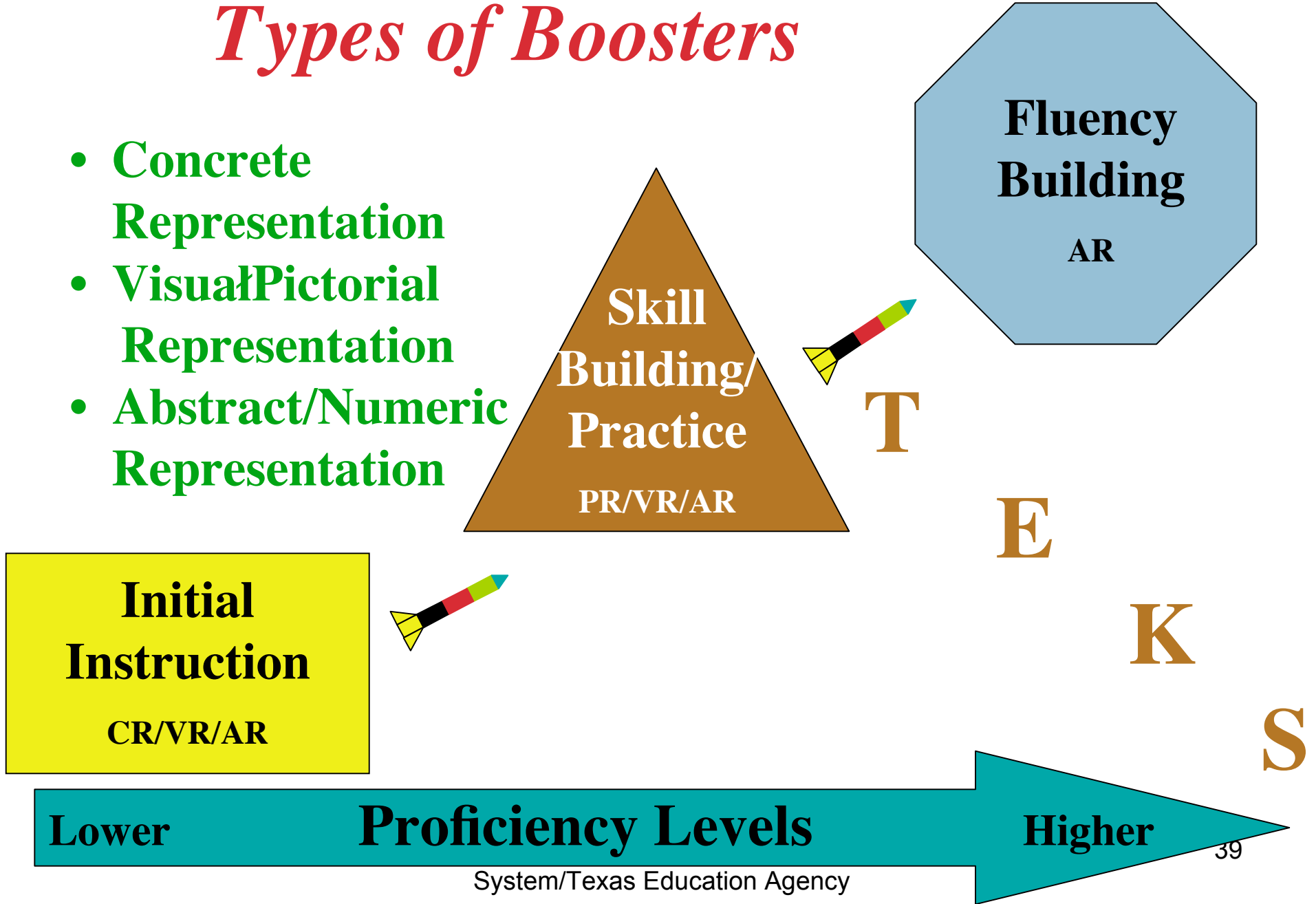
**Fluency Booster 1: Magnitude Comparisons**

**Warm-up: Counting to 20**

**Struggles with addition facts**

# Types of Boosters

- **Concrete Representation**
- **Visual/Pictorial Representation**
- **Abstract/Numeric Representation**



## **Features of Mathematics Interventions**

- (1) duration (daily; weekly) that can be integrated into an already crowded schedule that in today's classrooms includes a large instructional unit of time designated for reading and language arts instruction: We will implement intervention sessions 4 times per week for 20 minutes each session;**
- (2) logically developed interventions to show the linkage of the basic skills with problem solving application;**
- (3) instructional approach that focuses on how struggling students best respond to instruction: We will use the combined model of explicit and strategic instruction for basic skills and combinations, respectively;**
- (4) use of physical (concrete), visual (pictorial), and abstract (numbers) representations: We will scaffold instruction across the representations;**
- (5) manipulatives that help students to conceptualize mathematical skills and concepts including number charts, 5 - and 10-frames, counters, number lines, base -ten materials (Van de Walle, 2004 )**

## **Features of Mathematics Interventions (cont.)**

**(6) mathematics vocabulary (e.g., greater than/less than: We will integrate instruction on mathematics vocabulary in interventions;**

**(7) story problem structures (i.e., part -part-whole, join, separate, and compare) recommended by researchers (e.g., Riley, Greeno, & Heller, 1983) (to be developed): We will focus on teaching students to generalize basic skills learning to contextualized problems;**

**(8) building automaticity: We will incorporate fluency building activities into our lessons (e.g., “Fast Facts”); and**

**(9) scaffolding: We will develop booster interventions that incorporate the skills and concepts (e.g., Warm Up -start with counting backward, Numeric Sequencing, Magnitude Comparison, arithmetic combinations, story problem application.**

## Effective Instruction for Booster Sessions

**F**raming the lesson

**P**reviewing

**M**odeling w/think alouds

**G**uided practice

**I**ndependent practice

**C**hecking for understanding

**E**rror correction and feedback

**P**rogress monitoring

## What are helpful resources?

- Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal*, 103, 51-73.
- Chard, D., Clarke, B., Baker, B., Otterstedt, J., Braun, D., & Katz, R. (in press). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment Issues in Special Education*.
- Clark, B., & Shinn, M.R. (in press). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review*.
- Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37, 4-15.
- Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education*, 33, 18-28.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
- Special issue of JLD-July issue-Gersten & Jordan

Vaughn Gross Center for Reading and Language Arts

For a copy of this presentation and more information, go to

[www.texasreading.org](http://www.texasreading.org)

Research: Special Education

Current research projects: Three-Tier Math

LDAT 3-Tier;

[http://www.texasreading.org/utcrla/materials/serp\\_prereferral\\_booklet.asp](http://www.texasreading.org/utcrla/materials/serp_prereferral_booklet.asp)

Schwab

<http://www.schwablearning.org/resources.asp?g=4&s=1>