SELF-REGULATION STRATEGIES TO IMPROVE MATHEMATICAL PROBLEM SOLVING FOR STUDENTS WITH LEARNING DISABILITIES

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Abstract. This article provides a review of research in cognitive strategy instruction for improving mathematical problem solving for students with learning disabilities (LD). The particular focus is on one of the salient components of this instructional approach – self-regulation. Seven studies utilizing this approach for teaching problem solving to students with LD were previously evaluated to determine its status as evidence-based practice. The results of this evaluation are described, and the self-regulation component embedded in the cognitive routine for each of the studies is presented. The article concludes with a discussion of several principles associated with research and practice in strategy instruction and some practical considerations for implementation in schools.

This article provides a review of research in strategy instruction for improving mathematical problem solving for students with learning disabilities (LD) with a focus on one of the salient components of this instructional approach – self-regulation. Research has consistently shown that students with LD are poor self-regulators who benefit from strategy instruction that incorporates self-regulation training (Graham & Harris, 2003; Wong, Harris, Graham, & Butler, 2003).

Self-regulation, the ability to regulate one’s cognitive activities, underlies the executive processes and functions associated with metacognition (Flavell, 1976). Metacognition has to do with knowledge and awareness of one’s cognitive strengths and weaknesses as well as self-regulation, which guides an individual in the coordination of that awareness while engaged in cognitive activities (Wong, 1999). Self-regulation strategies, such as self-instruction, self-questioning, self-monitoring, self-evaluation, and self-reinforcement, help learners gain access to cognitive processes that facilitate learning, guide learners as they apply the processes within and across domains, and regulate their application and overall performance of a task.

Swanson’s (Swanson, 1999; Swanson & Sachs-Lee, 2000) meta-analyses of 30 years of both group and single-subject intervention studies conducted with students with LD revealed that direct instruction and strategy instruction were the two most effective instructional approaches, particularly when combined, for teaching students with LD across academic domains (i.e., reading, writing, and mathematics).

Interventions were considered direct instruction if they contained the following components: (a) drills and probes, (b) repeated feedback, (c) rapidly paced instruction, (d) individualized instruction, (e) breaking the task down into a sequence of steps, (f) pictorial diagrams, (g) small-group instruction, and (h) direct questioning by the teacher (Swanson, 1999).

In contrast, strategy instruction focuses on processes; for example, metacognition or self-regulation. The following procedures characterized strategy instruction: (a) systematic and direct explanations and/or verbal
were controlled, and established social importance and cost-effectiveness of the intervention

For the group-design studies, the benchmarks included (Gersten et al., 2005):

1. Research based on previous studies or a compelling argument for its importance
2. Sufficient description of the participants, setting, attrition, and intervention agents
3. Sufficient description of the intervention, procedures for determining fidelity of implementation, and differences between treatment and control groups
4. Sufficient description of the measures and technical adequacy and data collection procedures
5. Sufficient description of the analytic procedures with emphasis on the power analysis, unit of analysis, and variability in the sample

These studies were then reviewed using the benchmarks to determine the quality of the research and, ultimately, to draw conclusions as to whether cognitive strategy instruction is evidence-based or at least promising (Montague & Dietz, in press).

The remainder of this article provides a summary of the results of the review, describes the self-regulation component embedded in the cognitive routine for each of the studies, reviews several principles associated with research in strategy instruction, and offers some guidelines for implementation.

Results of the Literature Review

Montague and Dietz (in press) evaluated five single-subject studies: Montague and Bos (1986); Case, Harris, and Graham (1992); Montague (1992); Hutchinson (1993); and Cassel and Reid (1996); and two group studies: Montague, Applegate, and Marquard (1993); and Chung and Tam (2005). The studies were rated by three independent raters to determine (a) whether each study met each of the quality indicators listed above; (b) whether each individual study met the criteria for “high quality” research; and (c) whether, as a body of work, the research met the standards for deeming the practice “evidence-based.”

Single-subject design studies. For the single-design studies to meet the standards, the body of research must have included at least five studies that met minimally acceptable methodological criteria, documented experimental control, appeared in peer-reviewed journals, were conducted by at least three different researchers across at least three geographical locations, and had at least 20 participants across studies.

When applying the standards and criteria developed by Horner et al. (2005) to evaluate the quality of the research, the five single-subject design studies stood up well. All used researcher-developed interventions,
which, although similar in many respects, varied somewhat with regard to the cognitive and metacognitive components. All interventions produced positive outcomes for individual students. Performance improved, although some students did not meet the criterion for mastery. Most students showed maintenance over time and maintained use of the strategy in classroom settings. However, there was evidence that performance declined over time without distributed review and practice. An overall analysis of the studies as a group concluded that the practice – cognitive strategy instruction – is evidence-based and does improve mathematical problem solving for students with mathematical disabilities.

**Group-design studies.** For the two group-design studies to meet the standards, the body of research must have included at least four acceptable studies or two high-quality studies that supported the practice. In addition, to be considered evidence-based, the weighted effect size must have been significantly greater than zero; for “promising,” there must have been at least a 20% confidence interval for the weighted effect size that was greater than zero.

The two group studies did not meet the criteria for either evidence-based or promising practice due to methodological issues. The primary problems for both studies included a lack of procedures to measure treatment fidelity and limited information regarding the technical adequacy of the outcome measures. This suggests that group studies designed to test the effectiveness of this practice need to be more rigorous and designed with the quality indicators in mind. All raters agreed that the interventions for both studies were described clearly and the results were positive.

**Self-Regulation Components of Cognitive Strategy Instruction**

The goal of cognitive strategy instruction is to teach learners multiple cognitive and metacognitive processes and strategies to facilitate and enhance performance in academic domains (e.g., mathematical problem solving) as well as nonacademic domains (e.g., social problem solving). The processes and strategies range from simple to complex depending on task difficulty and context of the task.

Students with LD characteristically are poor strategic learners and problem solvers and manifest strategy deficits and differences that impede performance, particularly on tasks requiring higher level processing. These students need explicit instruction in selecting strategies appropriate to the task, applying the strategies in the context of the task, and monitoring their execution. They have difficulty abandoning and replacing ineffective strategies, adapting strategies to other similar tasks, and generalizing strategies to other situations and settings. Instruction aims to develop strategic learners who have an effective and efficient repertoire of strategies and are motivated, self-directed, and self-regulating.

In contrast to direct instruction, which is didactic and grounded in behaviorism, the theoretical foundation of cognitive strategy instruction considers both behavioral and cognitive theory; that is, information processing and developmental theory. Instruction focuses on cognitive processes, such as visualization, and metacognitive or self-regulation strategies, such as self-questioning. Cognitive strategy instruction teaches students to think and behave like good problem solvers and strategic learners. A cognitive routine is taught using explicit instruction, an instructional model that consists of very structured and organized lessons, appropriate cues and prompts, guided and distributed practice, cognitive modeling, interaction between teachers and students, immediate and corrective feedback on performance, positive reinforcement, over-learning, and mastery.

All the studies included in Montague and Dietz’s (in press) review focused on teaching a specific cognitive routine for mathematical problem solving that includes a self-regulation component. The studies included a total of 142 students ranging in age from 8-4 to 16-7 years. Most of the participants were identified with learning disabilities (N = 110), while two identified participants as having mild intellectual disabilities (Cassel & Reid, Chung & Tam, 2005). Montague used additional preset criteria for participation that included average intelligence, at least a third-grade reading level, and facility with the four basic math operations using whole numbers and decimals.

**Montague et al. (1986, 1992, 1993).** Montague’s cognitive routine (Montague & Bos, 1986; Montague, 1992; Montague et al., 1993) is a seven-phase model with specific self-regulation components. In the 1986 study, self-regulation was embedded in a script; for example, A self-questioning technique such as “What is asked?” or “What am I looking for?” was used to provide focus on the outcome.

The two later studies specified a SAY, ASK, CHECK routine for each of the seven processes taught. SAY requires students to self-instruct, which helps students identify and direct themselves as they solve the problem. For example, when reading the problem, students SAY “Read the problem. If I don’t understand it, read it again.” ASK refers to self-questioning, which promotes internal dialogue that helps to systematically analyze the problem information and regulate execution of the cognitive processes. When students paraphrase the problem, they ASK themselves, “Have I underlined the
important information? What is the question? What am I looking for?” Finally, CHECK is the self-monitoring strategy that promotes appropriate use of specific strategies and encourages students to monitor their performance throughout the problem solving process. When students formulate a visual representation of the problem, they CHECK “the picture against the problem information.”

Figure 1 presents the entire routine – the seven processes and the corresponding SAY, ASK, CHECK component for each. Students are required to memorize the processes and become familiar with the self-regulation component. After students understand what the processes are and can recite them from memory, the teacher uses process or cognitive modeling to demonstrate how good problem solvers approach a mathematical problem. Students are then required to “think aloud” as they solve practice problems. Finally, they become the “teacher,” modeling how good problem solvers think and behave.

In the two single-subject studies (Montague & Bos, 1986; Montague, 1992), the strategy use of six secondary and six middle school students improved substantially, and strategy maintenance was evident. However, the two sixth graders did not meet the mastery criterion, suggesting that the comprehensive cognitive routine may have been developmentally beyond their ability. For the group study, 72 middle school students were taught in groups of 8-12; on the posttest, they performed to the level of a group of nondisabled students.

Graham and Harris (2003). The Self-Regulated Strategy Development model (SRSD; Graham & Harris, 2003), designed in the early 1980s to improve composition skills of students with LD, was the basis for the intervention studies by Case et al. (1992) and Cassel and Reid (1996). This model includes the basic components of all cognitive strategy instructional routines. The model consists of six stages to guide instruction: (a) develop and activate background knowledge by providing the knowledge and skills needed to acquire and apply strategies and procedures for problem solving, (b) discuss the strategy by looking at the student’s current performance and explaining the strategies and how they will help the student improve their problem solving, (c) model the strategy using “think aloud” to demonstrate how giving oneself instructions helps regulate strategy use during problem solving, (d) have students memorize the strategy steps and self-statements, (e) support strategy use by providing guided practice using scaffolded instructional techniques, and (f) monitor students’ performance until they can use the specific math problem-solving and self-regulation strategies independently.

Case et al. (1992). The variation of the SRSD model in the study by Case et al. (1992) included preskill development; conferencing regarding each student’s current performance level, metastrategy information, and commitment to learning the strategy; discussing the problem-solving strategy; modeling the strategy and self-instructions; mastery of the strategy steps; collaboratively practicing the strategy and self-instructions; independent performance; and generalization and maintenance components. As part of the package, instructional goals were set collaboratively by the student and the teacher, followed by a discussion of the importance of the strategy and the self-regulation strategies (self-assessment, self-recording, and self-instruction).

The strategy was introduced using a small chart listing the following five steps:
1. Read the problem out loud.
2. Look for important words and circle them.
3. Draw pictures to help tell what is happening.
4. Write down the math sentence.
5. Write down the answer.

Four students with LD in grades 5 and 6 progressed from learning to apply the strategy with simple addition problems to subtraction problems. Students’ performance on the addition problems remained high after instruction. On the subtraction problems, student performance increased dramatically, and students were able to discriminate between addition and subtraction problems, thus minimizing selection of the wrong operation.

Cassel and Reid (1996). Cassel and Reid (1996) used similar procedures to teach the strategy: preskill development, initial conference, discussion of the problem-solving strategy and self-regulation procedures, modeling the strategy and self-instructions, strategy mastery, collaborative practice, independent practice, and maintenance.

The strategy consisted of the following nine steps and the acronym “FAST DRAW.”
1. Read the problem out loud.
2. Find and highlight the question, then write the label.
3. Ask what are the parts of the problem, then write the numbers needed.
4. Set up the problem by writing and labeling the numbers.
5. Reread the problem and tie down the sign (decide if you use addition or subtraction).
6. Discover the sign (recheck the operation).
7. Read the number problem.
8. Answer the number problem.
9. Write the answer and check by asking if the answer makes sense.

The teacher modeled strategy use using self-talk and self-questioning; for example, “What is it I have to do?”
Figure 1. Math problem-solving processes and strategies.

READ (for understanding)
Say: Read the problem. If I don’t understand, read it again.
Ask: Have I read and understood the problem?
Check: For understanding as I solve the problem.

PARAPHRASE (your own words)
Say: Underline the important information. Put the problem in my own words.
Ask: Have I underlined the important information? What is the question?
What am I looking for?
Check: That the information goes with the question.

VISUALIZE (a picture or a diagram)
Say: Make a drawing or a diagram. Show the relationships among the problem parts.
Ask: Does the picture fit the problem? Did I show the relationships?
Check: The picture against the problem information.

HYPOTHESIZE (a plan to solve the problem)
Say: Decide how many steps and operations are needed. Write the operation symbols (+, -, x, and /).
Ask: If I ..., what will I get? If I ..., then what do I need to do next? How many steps are needed?
Check: That the plan makes sense.

ESTIMATE (predict the answer)
Say: Round the numbers, do the problem in my head, and write the estimate.
Ask: Did I round up and down? Did I write the estimate?
Check: That I used the important information.

COMPUTE (do the arithmetic)
Say: Do the operations in the right order.
Ask: How does my answer compare with my estimate? Does my answer make sense?
Are the decimals or money signs in the right places?
Check: That all the operations were done in the right order.

CHECK (make sure everything is right)
Say: Check the plan to make sure it is right. Check the computation.
Ask: Have I checked every step? Have I checked the computation? Is my answer right?
Check: That everything is right. If not, go back. Ask for help if I need it.

“How can I solve this problem?” “FAST DRAW will help me organize my problem solving and remember all the things I need to do in order to successfully complete a word problem.” “Oops, I made a mistake, so I need to correct it.” Four third and fourth graders reached mastery on several types of addition and subtraction problems and maintained performance over time.

**Hutchinson (1993).** Hutchinson’s study (1993) targeted three types of algebra problems: relational problems, proportion problems, and two-variable two-equation problems. Twelve secondary school students were taught a strategy that included two types of self-regulation components. The first consisted of a series of self-questions for representing the problems and a second series of self-questions for solving the algebra problems. The self-questions for representing algebra word problems were as follows:

1. Have I read and understood each sentence? Are there any words whose meaning I have to ask?
2. Have I got the whole picture, a representation, for the problem?
3. Have I written down my representation on the worksheet? (goal, unknown(s), known(s), type of problem, equation)
4. What should I look for in a new problem to see it is the same kind of problem?

The self-questions for solving algebra word problems were:

1. Have I written an equation?
2. Have I expanded the terms?
3. Have I written out the steps of my solution on the worksheet? (collected like terms, isolated unknown(s), solved for unknown(s), checked my answer with the goal, highlighted my answer)
4. What should I look for in a new problem to see if it is the same kind of problem?

The second self-regulation component was a structured worksheet with the following prompts: (a) Goal, (b) What I don’t know, (c) What I know, (d) I can write/say this problem in my own words. Draw a picture, (e) Kind of problem, (f) Equation, (g) Solving the equation, (h) Solution, (i) Compare to goal, and (j) Check. Using this strategy, students improved substantially in algebra problem solving, had significantly higher posttest scores than a comparison group, and maintained performance over time.

**Chung and Tam (2005).** The last study, conducted by Chung and Tam (2005) with 30 Chinese students with mild intellectual disabilities, used a modification of Montague’s (1992) cognitive routine. The researchers’ variation included the following five steps:

1. Read the problem out loud.
2. Select the important information.
3. Draw a representation of the problem.
4. Write down the steps for doing the computation.
5. Check the answer.

The self-regulation component was an adaptation of Montague’s (1992) SAY, ASK, CHECK procedure (see Figure 1). Students were randomly assigned to (a) conventional instruction, (b) worked example instruction, or (c) cognitive strategy instruction. Students in the worked example and cognitive strategy instruction groups outperformed students receiving conventional instruction on immediate and delayed measures of two-step addition and subtraction problems.

In sum, these studies had a common goal: to improve mathematical problem solving for students with LD using an instructional approach that promotes strategic and self-regulated learning. Self-regulation is integral to cognitive strategy instruction as it directs and guides students in the application of the problem-solving process and is essential to effective and efficient mathematical problem solving.

The concluding sections of this article focus on guidelines for future research in strategy instruction and on practical considerations for implementing cognitive strategy instruction in today’s schools. Eight principles of instruction discussed by Swanson (1999), derived from the literature on cognitive, learning, and memory, serve as guidelines for implementing and evaluating cognitive strategy instruction and should be considered in future research investigating “evidence-based practices.” That is, interventions must be not only effective but also efficient, and teachers must consider the practices to be feasible and usable in typical classroom settings. Swanson’s principles provide guidance in selecting and implementing evidence-based practices.

**Principles of Strategy Instruction**

**Principle 1: Instruction must operate on the law of parsimony.** As discussed, most cognitive strategy instruction programs have multiple components (see, for example, Cassel & Reid, 1996). In essence, they are packages of content, strategies, and procedures. Determining the components of instruction that best predict student performance is a challenge for intervention researchers. Montague et al. (1993) attempted to address this question by separating the cognitive and metacognitive components in their routine and concluded that both were required, particularly for maintaining performance over time. Swanson’s review (1999) suggests that the best of the instructional programs include (a) teaching a few critical strategies well; (b) teaching students to monitor their learning and performance; (c) teaching students how, when, and where to use the strategies to promote generalization; (d) integrating strategy instruction into the general curricu-
lum; and (e) providing ongoing supervised student feedback and distributed practice.

**Principle 2. The use of effective instructional strategies does not necessarily eliminate processing differences in students.** In other words, research must include measures of both cognitive processes and strategies as well as academic measures and groups of students with and without LD to determine the impact not only on academic performance but also on the cognitive and metacognitive processes and strategies underlying performance. To illustrate, Hutchinson (1993) used a “think aloud” procedure and a metacognitive interview to ascertain growth in students' understanding and use of strategies during algebra problem solving.

**Principle 3: Instructional strategies serve different purposes.** By this, Swanson (1999) noted that certain components of instruction and combinations of components have a differential impact on performance in different domains. That is, in his synthesis, no one combination of instructional components was responsible for outcomes across domains.

**Principle 4: Comparable performance does not mean students use comparable processes or strategies.** Students with LD may perform as well as nondisabled students on some tasks but may use different processes or strategies to achieve the goal. These students may have a repertoire of strategies that may be effective on relatively simple tasks or tasks that present little difficulty, but on more complex activities, such strategies may not apply or may not be sufficient. Identifying the strategies students have and use appropriately and those they need to be successful on more difficult tasks is an additional challenge for intervention research.

A simple informal procedure like the Mathematical Problem Solving Assessment (Short Form) (Montague, 1992) provides information about students' perception of ability; attitude toward mathematics and math problem solving; and knowledge, use, and control of math problem-solving processes and strategies. Informal measures like this provide insight into students' knowledge of strategies and their ability to apply them appropriately on tasks like math problem solving that require higher order processing.

**Principle 5: Strategies must be considered in relation to a student's knowledge base and capacity.** Whether students will benefit from various types and levels of strategy instruction may depend on their cognitive characteristics such as intellectual ability or memory capacity. Thus, successful strategy instruction must consider the match between the strategy and learner characteristics. Therefore, assessing students prior to cognitive strategy instruction in a domain like mathematics is important to determine if they have the competencies to benefit from instruction as designed.

Otherwise, modifications to the cognitive routine and instructional procedures may be necessary. For example, Montague et al. (1993) excluded sixth graders from the group study because they had not met the criterion for mastery in the earlier single-subject study (Montague, 1992). The researchers concluded that sixth-grade students may not be maturationally ready for the comprehensive cognitive routine as designed and that the routine should be modified for younger learners.

**Principle 6: Comparable instructional procedures may not eliminate performance differences.** This relates to the idea that students with LD may learn to use a strategy as well as their nondisabled counterparts but may still not perform as well on an academic task. To reach the performance level of peers, some students with LD need additional intervention.

**Principle 7: Good instructional approaches for students with LD are not necessarily good approaches for nondisabled students and vice versa.** This principle is very important. The cognitive strategy instruction interventions described in this article were developed specifically for students with LD with knowledge of their cognitive and behavioral characteristics. It is important to remember that students with LD are not performing as well as their nondisabled peers for a variety of reasons, so the challenge for intervention researchers is to describe not only the characteristics of students but also how these characteristics interact with the components of cognitive strategy instruction.

**Principle 8: Instructional strategies as taught do not necessarily generalize to other situations, settings, and tasks.** Evidence suggests that as children acquire simple strategies, the strategies undergo modification or transformation as they are applied to other and more difficult tasks, thus allowing generalization of strategy use (Pressley, Brown, El-Dinary, & Allferbach, 1995). Students with LD may not possess the cognitive mechanisms to facilitate strategy transformation, or if they do, may fail to use the mechanisms appropriately to adapt and modify strategies to perform more efficiently. If students are expected to generalize strategy use to other situations, settings, and tasks, then instruction must include procedures to promote generalization.

**Implications for Practice**

In conclusion, cognitive strategy instruction to improve mathematical problem solving for students with LD appears to qualify as an evidence-based practice. The primary question regarding implementing cognitive strategy instruction is: How, when, and by whom should cognitive strategy instruction be provided for students with LD?
Let's first consider the ideal conditions. Instruction should be provided by expert remedial teachers who understand the characteristics of students with LD. Instruction should be provided to small groups of students (e.g., 8-10 students), who have been assessed to determine if they will benefit from instruction. Instruction should be intense and time-limited, so teachers may wish to remove students from the general education classroom for the duration of strategy instruction and include procedures to ensure that students will generalize strategy use after returning to the class. This requires collaboration between general and special education teachers.

However, the ideal may not be possible for several reasons. First, with the move toward inclusion in most districts, students with LD are being placed in general education mathematics classes often with teachers who have no or limited background teaching these students. Second, teachers may not have the necessary expertise or background in strategy instruction. Therefore, they may need professional development and continued support from a specialist to implement strategy instruction with fidelity. Third, teachers may be pressured by the district to complete the required curriculum and prepare students for state assessments. As a result, they may feel they do not have sufficient time to implement strategy instruction.

The above can be serious impediments to implementing evidence-based practices like cognitive strategy instruction for students with LD in typical classroom settings. Obtaining the support of district- and school-level administrators and the commitment of both general and special education teachers is critical to successful implementation of cognitive strategy instruction for students with LD.

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